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Dynamic bargaining with action-dependent valuations

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Abstract

An infinite-horizon, dynamic bargaining model is presented in which actions affect the expected future value of the buyer–seller match. Because actions directly affect the future surplus to be bargained over, the model is unlike other models that tie the dynamic process to nature alone. Focusing on a subset of weak Markov equilibria, several results come about that are not found in static bargaining models using similar bargaining protocols. In particular, optimal price demands can be lower (higher) than the buyer's lowest (highest) possible valuation, and several empirical features concerning wage settlements and strike incidence from labor union contract negotiations can be explained.

JEL classification: C73; C78; D82; J52

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1. Introduction

There are many situations in which a buyer and seller meet regularly to negotiate the sale of the seller's stock of a good or the seller's claim to a flow good, where the current surplus to be bargained over has been influenced by the previous negotiation. The dynamic bargaining model developed below is unique in that it allows for current delays in bargaining to negatively affect the expected future value of the surplus. Specifically, the longer negotiations currently continue without a settlement, the lower is the expected value of the buyer–seller match at the next negotiation. When strikes negatively affect the future expected surplus value, the buyer's and seller's maximization

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problems include a dimension not found in static bargaining models: namely, in addition to valuing current payoffs, both sides now care about how their actions and their opponent's actions affect future negotiations. The most notable example is that of a union and firm meeting every few years to negotiate the price at which the firm will purchase labor from the union, with strikes leading to the union–firm match becoming less valuable as strikes cause the firm to lose market position, high quality workers, good will, or customers.

Except for the dynamic nature of the game, the details of the bargaining problem presented below mirror those in the static bargaining literature. The surplus value, known only to the buyer, is newly drawn for each contract period.[1] Not knowing the surplus value, the seller makes at most two price demands during each negotiation that the buyer accepts or rejects. Allowing the seller two price demands instead of just one, as is more common in dynamic models, provides an environment in which strike duration can be analyzed. Following Fudenberg and Tirole (1983), Grossman and Perry (1986), Hart (1989), Sobel and Takahashi (1983) and others, the seller is not permitted to pre-commit to a sequence of demands.[2]

The subset of weak Markov equilibria that depends only on the current round's parameters and actions is investigated. At extreme parameter values, the seller always makes either soft or tough price demands despite the dynamic relationships across contracts. At less extreme parameter values, the seller chooses her demands (and the buyer accepts or rejects) depending on the effect delaying an agreement has on the future. In equilibrium, the seller plays more aggressively (by demanding higher prices) following an immediate agreement than following a strike, because the current expected surplus is higher the more quickly the previous negotiation was settled. Moreover, the degree to which the seller plays tough varies directly with the likelihood that the buyer's current valuation is high (and inversely with the length of the strike during the previous contract negotiation). As with similar static models, the seller's strategy during each round of negotiations is associated with her making an optimal number of screening demands before making a pooling demand.

Compared to what she would do in a static game, it is shown that the seller sometimes makes lower demands in the dynamic game, occasionally even at a price below her own valuation, in order to preserve the future expected surplus. At other times the seller exploits the buyer's desire to preserve the future expected surplus by making higher demands in the dynamic game than she would in the static game. The seller may even demand and the buyer accept a price above the buyer's highest possible valuation. These extreme price demands can never be optimal in static models or repeated games. In the setting here, however, both agents care about avoiding delays in order to increase the expected future surplus. When the seller offers below her own

[1] When motivated by labor negotiations, strikes negatively affect the future marginal revenue product of labor (MRPL). Moreover, the firm observes the MRPL whereas the union, having less information about capital and product markets, cannot observe the MRPL.

[2] Card (1990), Hayes (1984) and Tracy (1987) develop static bargaining models involving a union and firm in which the union can pre-commit to a sequence of wage demands. Sobel and Takahashi (1983) solve their model under both commitment and non-commitment. Kennan (1986) and Kennan and Wilson (1989, 1993) provide general discussions of commitment versus non-commitment in bargaining models.

valuation, she does so anticipating that the buyer will accept, thus making the expected future surplus high and allowing herself to extract more (expected) surplus from the buyer during the next negotiation. Similarly, when the buyer accepts a price above his valuation, it is because accepting such a price increases the expected future surplus but not enough to entice the seller to play tough at the next negotiation.

A particular strength of the model is its ability to explain empirical patterns in labor negotiations. First, accepted prices demands (e.g., the wage rate) can increase over the course of negotiations. This supports empirical work concerning the relationship between wage acceptances and strike duration by Card (1990), Riddell (1980) and Vroman (1984). Second, even though strikes directly reduce the expected future surplus value of the buyer–seller match, the probability of encountering a strike can be greater following a short strike than following an immediate agreement or a long strike. This supports Card's (1988) longitudinal analysis of strike activity in the U.S.

The paper is organized as follows. Section 2 discusses dynamic bargaining and further motivates why strikes negatively affect future negotiations. The model is made formal in Section 3. A process that determines a weak Markov equilibrium for any set of parameter values is presented in Section 4. General properties of weak Markov equilibria are presented in Section 5, while several properties of the optimal price paths are derived in Section 6. Section 7 discusses the relationship between strike incidence and previous strike duration before concluding remarks are offered in Section 8.

2. Dynamic bargaining

Under dynamic bargaining, in contrast to repeated bargaining, the stage game changes across time periods either in its strategy space, timing of moves, or payoffs. Furthermore, the dynamic process determining the stage game can arise in several ways. Nature may be at work changing the underlying parameters; agents' actions may affect the future bargaining conditions; or each player's current payoffs may affect the set of possible strategies available to them in the future.

Recently, Kennan (1995, 2001), Rustichini and Villamil (1996) and Vincent (1998) have modeled a dynamic relationship between a buyer and seller in which nature controls the dynamics. In these papers, the surplus value, either high or low and known only to the buyer, follows a stochastic process from one contract to the next. Unless the stochastic process is completely transitory or completely persistent, the seller offers a sequence of pooling and screening demands. When optimistic, the seller demands a high price. Following a rejection, the seller believes the valuation is low and makes several pooling demands over time, each of which the buyer accepts. The stochastic process is applied after each negotiation. Eventually the seller regains her optimism that the valuation is high and screens again.

Unlike these previous papers, the dynamics in the model analyzed in this paper are driven by actions. As both agents observe the other's actions, there is no *useful* private information *across contracts*. Rather, at the start of each contract period, nature draws the surplus for that contract where the length of delay during the previous negotiation determines the expected value of the surplus. In particular, the current expected surplus is greater the faster the two parties previously reached agreement. Thus, the buyer directly affects the expected future surplus with each of his acceptance/rejection decisions, and the seller indirectly affects the expected future surplus by demanding low or high prices.

In many bargaining situations, assuming that current delays negatively affect the conditions under which future bargains take place is reasonable. This relationship is fundamental in labor negotiations involving a union and a firm. Rees (1977) acknowledges the negative impact labor strikes may have on the future value of the union–firm match by noting that a strike likely causes the firm to permanently lose some of its customers and highest quality workers to its competitors. Firms also develop connections to sell their products to the same factories, distributors, and/or consumers. A strike puts tension on these connections and may lead to the termination of this channel of business for the firm. If a significant portion of a firm's clientele is lost during a strike, the effects of a strike will remain well after the strike has been settled as the firm tries to rebuild the broken connections and reclaim its clientele base. Labor disputes at Continental Airlines (1983), Eastern Airlines (1989–90), the *New York Daily News* (1990), Greyhound Bus Company (1990–91), and Major League Baseball (1994–95) all resulted in decreased patronage for several years (and even the demise of the firm in the case of Eastern Airlines).

3. The model

Let $t = 0, 1, 2, \ldots$ index contract periods. A buyer and a seller, both of whom are risk-neutral and discount future contract periods at rate δ, $0 < \delta < 1$, are matched forever. Each period the seller possesses one unit of a perishable, indivisible good that she values at 0. The buyer's valuation for the good is either v_L or v_H with $0 < v_L < v_H$. For convenience, let $v_H - v_L$ be the unit of measurement, and let θ represent the buyer's low valuation in terms of the new units: $\theta = v_L/(v_H - v_L)$. Finally, let v_t represent the buyer's valuation for the good over contract period t in the new units so that $v_t \in \{\theta, \theta + 1\}$.

As the seller makes at most two price offers during each contract negotiation and the buyer decides whether to accept these offers, negotiations for contract t are associated with up to four actions: $p_{t,1}$, $q_{t,1}$, $p_{t,2}$, and $q_{t,2}$ where p indicates a price offer, q indicates an acceptance or rejection (i.e., the quantity traded), and the second subscript refers to the first or second offer.

With the buyer knowing v_t and the seller knowing the probability that v_t equals θ, denoted by ζ_t, negotiations during each contract period proceed as follows. The seller demands a price of $p_{t,1}$. If the buyer accepts this demand ($q_{t,1} = 1$), he pays the seller the stated price in exchange for the good. If the buyer rejects the demand ($q_{t,1} = 0$), the seller demands a second and final price of $p_{t,2}$. Delay between the first and second demands results in a portion of the good being lost. Although this loss can be from discounting, in the context of a union and firm, the loss can also result from lost production time. A firm operating at full capacity, for example, cannot recoup lost production time after an agreement is eventually reached. Let k be the proportion of the surplus remaining when the buyer accepts or rejects the second demand. If the buyer rejects the first demand but accepts the second ($q_{t,1} = 0,\ q_{t,2} = k$), he pays the seller $kp_{t,2}$ in exchange for the remaining portion of the good which he now values at kv_t. If the buyer rejects both price demands ($q_{t,1} = q_{t,2} = 0$), the good perishes with nothing traded during the current contract period. The game then proceeds to period $t+1$ at which time v_{t+1} is revealed to the buyer and the negotiation process begins again.

Because the seller is given at most two price demands, each contract's negotiation results in no strike if the buyer accepts the first price demand, a short strike if the buyer rejects the first but accepts the second demand, or a long strike if the buyer rejects both demands.[3] Define a_t as

$$a_t = \begin{cases} N & \text{if } q_{t,1} = 1 \text{ (i.e. negotiations for contract } t \text{ resulted in no strike)}, \\ S & \text{if } q_{t,2} = k \text{ (i.e. negotiations for contract } t \text{ resulted in a short strike)}, \\ L & \text{if } q_{t,2} = 0 \text{ (i.e. negotiations for contract } t \text{ resulted in a long strike)}. \end{cases} \quad (1)$$

Of fundamental importance is the process controlling how the buyer's expected valuation changes from one contract to the next. This process is best thought of in the following way. At the start of contract period t, nature chooses n_t from an urn containing zeros and ones with the interpretation being that $v_t = \theta + n_t$.[4] Let ζ_t be the probability that a random draw by nature returns 0 so that ζ_t increases as the seller becomes more pessimistic about the buyer's valuation. The dynamic process of the bargaining game is completely captured by the relationship between a_{t-1} and ζ_t:

$$\zeta_t = \begin{cases} \zeta_N & \text{if } a_{t-1} = N, \\ \zeta_S & \text{if } a_{t-1} = S, \\ \zeta_L & \text{if } a_{t-1} = L, \end{cases} \qquad \text{where } 0 < \zeta_N < \zeta_S < \zeta_L < 1 \text{ and } t = 1, 2, 3, \ldots\ . \quad (2)$$

Notice the buyer's expected valuation is greatest when there was no strike during the previous negotiation and is least when there was a long strike during the previous negotiation.

A comment on the extensive form of the bargaining game is warranted. As delays in bargaining are costly, in terms of both current surplus and future expected surplus, long-term or infinitely lived contracts would increase efficiency and could possibly be welcomed by both parties. In general, bounded rationality vis-à-vis being unaware of future contingencies or there being too many contingencies to specify provides

[3] Granting the seller only two price demands during each negotiation is the simplest version of the model that allows for varying strike durations. Although a long strike is formally associated with no agreement during the current contract period, the model can be extended to allow for a third price demand. Parameter restrictions could then be imposed to force an agreement at that stage by limiting the joint value of all future negotiations if the strike continues. Thus, the appearance of a long strike as being one that is never settled is not necessary nor a desirable view of the model.

[4] Similarly, nature could be viewed as choosing $v_t \in \{\theta, \theta + 1\}$.

reasonable grounds for not considering indefinite contracts. Writing costs may also preclude long-term contracts.[5]

While negotiating contract t, there are four histories of importance. When choosing her first price demand, the seller knows all previous price offers and acceptance/rejection decisions of the buyer. In addition to knowing what the seller knows and being confronted with $p_{t,1}$, the buyer also knows the current and all previous surplus values, $\{n_1, n_2, \ldots, n_t\}$. If the buyer rejects $p_{t,1}$, the seller makes a second and final demand further knowing that the buyer rejected $p_{t,1}$. In deciding to accept or reject $p_{t,2}$, the buyer continues to know all current and previous prices, quantities, and surplus values.

Strategies for the buyer and seller, σ^{B} and σ^{S}, map histories into probability distribution spaces over respective action spaces. Let Σ^{B} and Σ^{S} be the buyer's and seller's set of all possible strategies.[6] Any σ^{B} in Σ^{B} paired with any σ^{S} in Σ^{S} is called a strategy profile, σ. Let $\mu_{t,1}$ and $\mu_{t,2}$ be the seller's belief that the buyer's valuation is low when she makes her first and second price demands respectively during negotiations for contract t. A belief system, denoted by μ, is a mapping from histories into beliefs.

In general there is no definition of sequential or perfect Bayesian equilibrium for dynamic games with infinite strategy sets. However, Kennan's (2001) extension of sequential optimality and consistency from Kreps and Wilson (1982) and Fudenberg and Tirole (1991) suffices. Sequential optimality requires strategies to yield (weakly) more expected surplus than any other possible strategy while holding fixed the other player's strategy. Consistency requires that (i) the seller's initial belief abide by the fundamental probability structure associated with the bargaining dynamics from Eq. (2), i.e., $\mu_{t,1} = \zeta_t$, and (ii) whenever possible, the seller update her beliefs according to Bayes' rule.[7] The only situation in which the seller cannot update using Bayes' rule is when $p_{t,1}$ is expected to be accepted by the buyer regardless of his valuation yet the buyer rejects. To address this possibility, Cho's (1987) introspective consistency is imposed. Under introspective consistency, the seller updates to believe that the buyer's valuation is low whenever the buyer rejects an offer that both buyer types were expected to accept.[8] A strategy-belief pair (σ, μ) is an equilibrium if it is sequentially optimal, consistent, and satisfies introspective consistency.

[5] For a discussion of the incomplete contract literature, see Anderlini and Felli (1999), Hart and Moore (1999), Maskin and Tirole (1999), Segal (1999) and Tirole (1999).

[6] The seller's strategy associates with any possible seller history a probability distribution over the real line from which a particular price demand is selected. Likewise, the buyer's strategy associates with any possible buyer history a probability distribution over the binary set $\{A, R\}$. The buyer then accepts or rejects according to this distribution. Histories, strategies, and beliefs are spelled-out completely in Lemke (2000).

[7] The Kreps and Wilson (1982) definition of consistency for finite games also requires that one be able to rationalize beliefs following zero probability events as the limit of a sequence of strategy-belief pairs so that each strategy in the sequence is fully determined by Bayes' rule. This requirement is satisfied for the strategies considered in the paper.

[8] Introspective consistency encompasses the intuitive criterion (Cho and Kreps, 1987) and, when the intuitive criterion is not applicable, assigns full weight to the buyer type having the least to lose from the ill-advised rejection.

4. Weak Markov perfect equilibrium

Without further restrictions, the dynamic nature of the game and the infinite choice sets leads to a multiplicity of equilibria. A standard refinement is to consider Markov perfect equilibria (MPE). MPE are formed by strategies using only payoff relevant elements of each agent's history, including the current state known by both parties. For the model here, Markov strategies require the seller's price demands to depend only on her belief of the buyer's valuation and the buyer's accept/reject decisions to depend only on the realized surplus value and the offered price.

As shown by Maskin and Tirole (1994), however, a necessary randomization off the equilibrium path precludes the existence of MPE in this type of game.[9] In lieu of MPE, attention will be focused on a particular set of weak Markov equilibria (WME) that allows strategies to depend on recent information that is no longer payoff relevant. In particular, off the equilibrium path, the seller will be allowed to condition $p_{t,2}$ on $p_{t,1}$. Formally, the seller's weak Markov strategies are all elements of Σ^{S} that satisfy

$$\sigma^{\mathrm{S}}_{t,1}\colon \zeta_t \to \Phi \quad \text{and} \quad \sigma^{\mathrm{S}}_{t,2}\colon \{(\mu_{t,2}, p_{t,1})\} \to \Phi, \tag{3}$$

where Φ is the set of all probability distributions over the real line. The buyer's weak Markov strategies are all elements of Σ^{B} that satisfy

$$\sigma^{\mathrm{B}}_{t,1}\colon \{(\zeta_t, n_t, p_{t,1})\} \to [0,1] \quad \text{and} \quad \sigma^{\mathrm{B}}_{t,2}\colon \{(n_t, p_{t,2})\} \to [0,1]. \tag{4}$$

Because the seller's first demand depends only on ζ_t, her behavior is similar to that in an analogous static bargaining game[10] in which the equilibrium involves the seller pooling, screening once and then pooling, or screening twice.[11] That is, although $\sigma^{\mathrm{S}}_{t,1}$ in Eq. (3) can associate a density function with each ζ_t, the optimal $\sigma^{\mathrm{S}}_{t,1}$ selects a degenerate distribution (i.e., a specific price) for each ζ_t and these prices correspond to pooling, screening once, or screening twice.

[9] On the equilibrium path, the seller updates her belief of the buyer's valuation to $\mu_{t,2}$ based on the rejection of $p_{t,1}$. In this way, $p_{t,1}$ is payoff relevant indirectly through its effect on $\mu_{t,2}$. If, however, the seller deviates by demanding an initial price between the low and high screening prices, the only way in which the strategies remain optimal in the face of this deviation is for the seller to condition her second price demand on her first. That is, following such a deviation, the seller not only updates her belief knowing $p_{t,1}$ was rejected but also needs $p_{t,1}$ to directly enter her strategy for choosing $p_{t,2}$ so that the buyer's randomization in accepting or rejecting $p_{t,1}$ was optimal. This requirement precludes the existence of a MPE. See Maskin and Tirole (1994) for a further discussion.

[10] Throughout the paper, the dynamic game will be compared to the 'static' game and the 'myopic' game. In the static game, the buyer and seller negotiate a single contract with $\zeta = \zeta_t$ given. The myopic game is identical to the dynamic game except that both agents completely discount future contracts, $\delta = 0$. That is, the players are myopic across contract periods but not within a contract period. Appendix A describes both games more thoroughly and solves for a perfect Bayesian equilibrium.

[11] Hart (1989) demonstrated that the structure of the equilibrium will always contain pooling and/or screening prices whenever the seller makes all of the price demands and there are a finite number of buyer valuations. In the dynamic game, the same behavior exists when strategies are Markov, because the buyer enters every negotiation with the same bargaining position (namely, the continuation value associated with rejecting both price demands) regardless of previous actions.

Claim 1. *When restricted to weak Markov strategies, the seller pools, screens once, or screens twice on the equilibrium path.*

Proof. See Appendix C. □

Notice that one can view the seller's strategy as having two parts: first the seller chooses to pool, screen once, or screen twice, and then the she chooses the highest prices that let her do this. The part of the decision that specifies pooling, screening once, or screening twice without regard for the actual prices is termed the seller's pricing plan.

Definition (Seller's pricing plan). The seller's pricing plan, ξ, is a three-tuple with each entry corresponding to pooling, screening once, or screening twice following no strike, a short strike, or a long strike:

$$\xi = (\xi_N, \xi_S, \xi_L) \quad \text{where } \xi_a \in \{0, 1, 2\} \text{ and } a \in \{N, S, L\}. \tag{5}$$

The interpretation of ξ is that when she must choose her first price demand, $\xi = (\xi_N, \xi_S, \xi_L)$ specifies how many screening demands the seller makes before pooling in each of the possible three states of the world. Let Ξ be the set of all 27 possible ξ's satisfying Eq. (5).

Prices are the only element now missing from the seller's strategy. Denote the five prices by P_{10}, P_{11}, P_{12}, P_{20}, and P_{21} where $P_{10} \leqslant P_{11} \leqslant P_{12}$ and $P_{20} \leqslant P_{21}$. When the seller pools, she demands $p_{t,1} = P_{10}$ (and $p_{t,2} = P_{20}$ although the buyer accepts $p_{t,1} = P_{10}$ in equilibrium). When the seller screens once, she demands $p_{t,1} = P_{11}$ and if rejected counters with the pooling price of $p_{t,2} = P_{20}$. When the seller screens twice, she demands $p_{t,1} = P_{12}$ and if rejected screens again by demanding $p_{t,2} = P_{21}$.

The buyer's weak Markov strategy and the seller's weak Markov strategy and belief system can now be stated formally.

Definition (Buyer's weak Markov strategy (BWMS)). The low valuation buyer (n_t=0):

$$q_{t,1} = \begin{cases} 1 & \text{if } p_{t,1} \leqslant P_{10}, \\ 0 & \text{if } p_{t,1} > P_{10}, \end{cases} \qquad q_{t,2} = \begin{cases} k & \text{if } p_{t,2} \leqslant P_{20}, \\ 0 & \text{if } p_{t,2} > P_{20}. \end{cases}$$

The high valuation buyer ($n_t = 1$):

$$q_{t,1} = \begin{cases} 1 & \text{if } p_{t,1} \leqslant P_{11}, \\ \begin{array}{l} 1 \text{ with prob. } 1-\rho_a \\ 0 \text{ with prob. } \rho_a \end{array} & \text{if } P_{11} < p_{t,1} \leqslant P_{12}, \\ 0 & \text{if } P_{12} < p_{t,1}, \end{cases} \qquad q_{t,2} = \begin{cases} k & \text{if } p_{t,2} \leqslant P_{21}, \\ 0 & \text{if } p_{t,2} > P_{21}, \end{cases}$$

where ρ_a is a randomization parameter discussed below.

If the buyer's valuation is low, he accepts any initial price at or below P_{10} and any second price at or below P_{20}. If the buyer's valuation is high, he accepts any initial price at or below P_{11}, rejects any price above P_{12}, and randomizes if the first demand is between P_{11} and P_{12}. When facing the seller's second demand, a high valuation buyer accepts if the demand is at most P_{21} and rejects otherwise.

Definition (Seller's weak Markov strategy (SWMS)). For $\xi \in \Xi$:

$$p_{t,1} = \begin{cases} P_{10} & \text{if } \xi_a = 0, \\ P_{11} & \text{if } \xi_a = 1, \\ P_{12} & \text{if } \xi_a = 2, \end{cases} \qquad p_{t,2} = \begin{cases} P_{20} & \text{if } \mu_{t,2} = 1, \\ \begin{matrix} P_{20} \text{ with prob. } 1-\gamma \\ P_{21} \text{ with prob. } \gamma \end{matrix} & \text{if } \zeta_t < \mu_{t,2} < 1, \\ \begin{matrix} P_{20} \text{ if } \zeta_t > \eta \\ P_{21} \text{ if } \zeta_t \leqslant \eta \end{matrix} & \text{if } \mu_{t,2} = \zeta_t, \end{cases}$$

where γ is a randomization parameter and η is a threshold value that will be discussed shortly.

The seller makes her first price demand according to her pricing plan, ξ. Following a rejection on the equilibrium path, the seller updates her belief of the buyer's valuation and then demands $p_{t,2}$ (also according to ξ). Off the equilibrium path, however, the seller updates her belief of the buyer's valuation and then demands $p_{t,2}$ accordingly.

Definition (Seller's belief system (SBS)).

$$\mu_{t,1} = \zeta_t; \qquad \mu_{t,2} = \begin{cases} 1 & \text{if } p_{t,1} \leqslant P_{11}, \\ \dfrac{\zeta_t}{\zeta_t + (1-\zeta_t)\rho_t} & \text{if } P_{11} < p_{t,1} \leqslant P_{12}, \\ \zeta_t & \text{if } P_{12} < p_{t,1}. \end{cases}$$

Suppose $p_{t,1}$ was rejected. According to *SBS*, if $p_{t,1}$ was less than or equal to P_{11}, the seller updates to believe the buyer has the low valuation as required under introspective consistency. If $p_{t,1}$ was greater than P_{11} but less than or equal to P_{12}, the seller updates according to Bayes' rule. If $p_{t,1}$ was greater than P_{12}, the seller retains her prior belief as both buyer types are expected to reject such a high initial price.

Definition (Weak Markov equilibrium (WME)). A strategy profile, σ, and a belief system, μ, form a WME if σ and μ satisfy *BWMS*, *SWMS*, *SBS*, consistency, sequential optimality, and introspective consistency.

Although a WME is defined in terms of the buyer's and seller's strategies listed above, the requirements of consistency and sequential optimality require a WME to also be a Nash equilibrium, i.e., it is an equilibrium in the presence of all possible unilateral deviations.

Let $\sigma = (\sigma^{\mathrm{B}}, \sigma^{\mathrm{S}})$ be any strategy profile such that σ^{B} abides by *BWMS* and σ^{S} abides by *SWMS*. Denote the seller's and buyer's expected lifetime discounted payoffs as they enter into negotiating a contract with strategy profile σ when the current state is ζ_a

where $a \in \{N, S, L\}$ by $U(\sigma|a)$ and $V(\sigma|a)$, respectively so that $U(\sigma|a)$ and $V(\sigma|a)$ are calculated before nature reveals the valuation. Denote the joint continuation value by $J(\sigma|a) = U(\sigma|a) + V(\sigma|a)$.

Let $\omega = \{\theta, k, \delta, \zeta_N, \zeta_S, \zeta_L\}$ and $\Omega = [0, \infty) \times (0, 1) \times (0, 1) \times \Gamma)$ where $\Gamma = \{(z_1, z_2, z_3): 0 < z_1 < z_2 < z_3 < 1\}$ so that Ω represents the subspace of $\mathbb{R}^6$ from which the parameters of the model can be chosen. For any $\omega \in \Omega$ and $\xi \in \Xi$, the five threshold prices and five randomization parameters can be found in order to ensure $BWMS$ is sequentially rational and consistent and that $SWMS$ is consistent. Sequential optimality from the seller's perspective can fail either because the seller is approaching negotiations with a sub-optimal pricing plan ξ, (e.g., the seller is pooling when she should screen once) or the seller is not extracting as much surplus as possible from the buyer (i.e., the seller could demand a higher price without affecting the buyer's decision). Avoiding this second condition requires that the seller's strategy incorporate tight prices, i.e., the prices most favorable to the seller such that the buyer willingly abides by $BWMS$. When using tight prices, therefore, checking sequential optimality for the seller only requires comparing the seller's payoffs across different choices of ξ.

When presented with a price demand, the buyer weighs his current and expected discounted future payoffs from acceptance against that from rejection. Thus, tight prices must depend on the buyer's but not on the seller's continuation values. For a given strategy profile, the optimal tight prices, the derivations of which are contained in Appendix B, are

$$
\begin{aligned}
&P^*_{10}(\sigma) = \theta + \delta[V(\sigma|N) - V(\sigma|L)], \\
&P^*_{11}(\sigma) = P^*_{10}(\sigma) + 1 - k, \quad P^*_{20}(\sigma) = \theta + \delta[V(\sigma|S) - V(\sigma|L)]/k, \\
&P^*_{12}(\sigma) = P^*_{10}(\sigma) + 1, \quad P^*_{21}(\sigma) = P^*_{20}(\sigma) + 1.
\end{aligned}
\tag{6}
$$

Appendix B also solves for the buyer's continuation values in terms of $V(\sigma|L)$:

$$
V(\sigma|a) = \begin{cases} 1 - \zeta_a + \delta V(\sigma|L) & \text{if } \xi_a = 0, \\ (1 - \zeta_a)k + \delta V(\sigma|L) & \text{if } \xi_a = 1, \\ \delta V(\sigma|L) & \text{if } \xi_a = 2. \end{cases}
\tag{7}
$$

And the joint continuation values, written recursively, follow immediately from $SWMS$ and $BWMS$:

$$
J(\sigma|a) = \begin{cases} 1 - \zeta_a + \theta + \delta J(\sigma|N) & \text{if } \xi_a = 0, \\ (1 - \zeta_a)[1 + \theta + \delta J(\sigma|N)] + \zeta_a[k\theta + \delta J(\sigma|S)] & \text{if } \xi_a = 1, \\ (1 - \rho_a)(1 - \zeta_a)[1 + \theta + \delta J(\sigma|N)] & \\ \quad + \rho_a(1 - \zeta_a)[k(1 + \theta) + \delta J(\sigma|S)] + \zeta_a \delta J(\sigma|L) & \text{if } \xi_a = 2. \end{cases}
\tag{8}
$$

The seller's continuation valuation is then the difference between the joint and the buyer's valuation:

$$
U(\sigma|a) = J(\sigma|a) - V(\sigma|a) \quad \text{for } a \in \{N, S, L\}.
\tag{9}
$$

It remains to find values for ρ_N, ρ_S, ρ_L, γ, and η so that the buyer and seller will not deviate from σ^{B} and σ^{S}. It is only when $n_t = 1$ and $P^*_{11} < p_{t,1} \leqslant P^*_{12}$ that the buyer randomizes his acceptance/rejection of the first price demand. Equating the expected lifetime payoffs from demanding P^*_{11} and P^*_{12} and then solving for γ yields

$$\gamma = [p_{t,1} - P^*_{11}]/k. \tag{10}$$

Notice that γ is in the unit interval as $P^*_{12} - P^*_{11} = k$ and $P^*_{11} < p_{t,1} \leqslant P^*_{12}$. Further, $\gamma = 0$ when $p_{t,1} = P_{11}$ so that the seller chooses $p_{t,2} = P_{20}$ following a rejection of the low screening price and $\gamma = 1$ when $p_{t,1} = P_{21}$ so that the seller chooses $p_{t,2} = P_{21}$ following a rejection of the high screening price.

If the buyer rejects $p_{t,1}$ when $P^*_{11} < p_{t,1} \leqslant P^*_{12}$, the seller weighs her expected payoff from pooling against that of screening. Setting these equal and substituting for $\mu_{t,2}$ which depends on ρ_a yields

$$\rho_a = \left(\frac{\zeta_t}{1-\zeta_t}\right)(\theta + (\delta/k)[J(\sigma|S) - J(\sigma|L)]).\ ^{12} \tag{11}$$

Finally, consider the case when $p_{t,1}$ was set above P^*_{12} so that $\mu_{t,2}$ equals ζ_t. Equating the seller's lifetime expected payoff from pooling to that from screening with her second demand and solving for ζ_t yields the threshold value denoted by η in $SWMS$:

$$\eta = \frac{k}{k + k\theta + \delta[J(\sigma|S) - J(\sigma|L)]}. \tag{12}$$

Although an analytical solution is not available, generating examples is straightforward. For any $\omega \in \Omega$, Eqs. (6)–(12) can be solved for any $\xi \in \Xi$. It then remains to check if the seller can gain by deviating. (Recall that tight prices guarantee the buyer cannot gain by deviating.) Although the seller can deviate to any price in $\mathbb{R}$, only a few deviations need to be checked, because the buyer's strategy is stated in terms of threshold prices. For example, suppose the seller considers deviating with her second demand. If the deviation is at most P^*_{20}, the buyer accepts. If the seller's deviation is more than P^*_{20} but no more than P^*_{21}, the buyer accepts if his valuation is high. And if the seller's deviation is greater than P^*_{21}, the buyer rejects. Therefore, one only needs to compare the seller's payoffs under compliance to her payoffs from demanding P^*_{10}, P^*_{11}, P^*_{12}, or any price above P^*_{12} with her first demand or from demanding P^*_{20}, P^*_{21}, or any price above P^*_{21} with her second demand.

Example. Suppose $\theta = 1.5$, $\delta = 0.8$, and $k = 2/3$. Further, assume that $\zeta_N = \zeta_L/4$ and $\zeta_S = \zeta_L/2$ so that ζ_L remains the only free parameter. The WME can then be determined

[12] If $\rho_a > 1$, screening twice is not credible for the seller. If the seller were to offer the high screening price, a high valuation buyer would reject and even knowing this the seller would optimally counter with a pooling demand.

as a function of ζ_L. Eq. (13) summarizes the seller's optimal pricing plan:

$$\xi^* = \begin{cases} \xi(2,2,2) & \text{if } 0.0000 < \zeta_L \leqslant 0.1667, \\ \xi(2,2,1) & \text{if } 0.1667 < \zeta_L \leqslant 0.3120, \\ \xi(2,2,0) & \text{if } 0.3120 < \zeta_L \leqslant 0.4440, \\ \xi(2,1,0) & \text{if } 0.4440 < \zeta_L \leqslant 0.7425, \\ \xi(1,0,0) & \text{if } 0.7425 < \zeta_L \leqslant 1.0000. \end{cases} \tag{13}$$

The seller always screens twice regardless of previous strike duration if ζ_L is low enough. As ζ_L increases (increasing ζ_S and ζ_N as well), the seller plays less tough, and eventually, when $\zeta_L > 0.7425$, she screens once following an immediate agreement and pools following either a short or long strike. [13]

5. General properties of WME

In the static or myopic version of the game, the seller plays less aggressively as the buyer's expected valuation decreases (see Appendix A). Theorem 1 shows the same equilibrium behavior exists in the dynamic game.

Theorem 1. *For any $\omega \in \Omega$, if (σ^*, μ^*) is a WME associated with a pricing strategy of ξ, then $\xi \in \Xi^*$, where $\Xi^* = \{(\xi_N, \xi_S, \xi_L)\colon \xi_N, \xi_S, \xi_L \in \{0,1,2\}$ and $\xi_N \geqslant \xi_S \geqslant \xi_L\}$.*

Proof. See Appendix C. □

For example, $\xi = (0,1,2)$ is not in Ξ^* (and therefore can never be a WME), because it requires the seller to be more aggressive following a long strike than she is following an immediate agreement (i.e., it requires that she pool after an immediate agreement, to screen once after a short strike, and to screen twice following a long strike).

In order to illustrate the claims in this section and the next, Tables 1–4 provide 15 numerical examples of WME. [14] Table 1 gives 10 examples of WME, one for each $\xi \in \Xi^*$. For each of the 10 examples, $\theta = 1.5$ and $\delta = 0.8$. Thus, the bad state is 60 percent worse than the good state, and discounting across contracts is on the order of signing two to three year contracts. For the 10 examples listed in Table 1, $k = (\zeta_L - \zeta_S)/(\zeta_L - \zeta_N)$. Thus, as the damage following a short strike approaches the damage following a long strike, k approaches 0, indicating that the second price demand is made late in the negotiation period. Similarly, if the expected surplus following a short strike is almost as great as the expected surplus following an immediate agreement, k is close to 1; the interpretation being that the second price demand is made soon after the first. The ζs

[13] A seller is said to be 'less aggressive' or to 'play softer' when she is more likely to pool than to screen (or more likely to screen once instead of twice) at the start of a contract period. Likewise, a 'more aggressive' or 'tougher' seller is more likely to screen than to pool (or to screen twice instead of once) at the start of a negotiation.

[14] All numerical examples were carried out in Maple V and are available from the author upon request.

Table 1
Examples of weak Markov equilibria

	Example 1.1: $\xi=(0,0,0)$			Example 1.2: $\xi=(1,0,0)$			Example 1.3: $\xi=(2,0,0)$			Example 1.4: $\xi=(1,1,0)$			Example 1.5: $\xi=(2,1,0)$		
a	N	S	L	N	S	L	N	S	L	N	S	L	N	S	L
ζ_a	0.50	0.70	0.90	0.30	0.60	0.90	0.05	0.65	0.95	0.30	0.35	0.40	0.15	0.35	0.55
$p_{t,1}$	1.82	1.82	1.82	2.2	1.7	1.7	2.46	1.46	1.46	1.8	1.8	1.3	2.14	1.64	1.14
$p_{t,2}$	1.82	1.82	1.82	1.98	1.98	1.98	3.22	2.22	2.22	1.06	1.06	1.06	2.3	1.3	1.3
$U(\sigma\|a)$	9.1	9.1	9.1	9.0524	8.9419	8.9419	10.503	9.8623	9.8623	7.0156	6.9495	6.9125	7.5590	7.2716	7.1872
$V(\sigma\|a)$	0.9	0.7	0.5	0.75	0.8	0.5	0.2	0.55	0.25	2.75	2.725	3	1.8	2.125	2.25
Myopic-U	7.5	7.5	7.5	8.0040	7.9032	7.9032	10.909	10.227	10.227	8.0469	7.9818	7.9375	8.8116	8.5341	8.5493
Myopic-V	2.5	2.3	2.1	1.7984	1.8387	1.5387	0.10455	0.43364	0.13364	1.7188	1.6927	1.975	0.49361	0.80789	0.84489

	Example 1.6: $\xi=(1,1,1)$			Example 1.7: $\xi=(2,1,1)$			Example 1.8: $\xi=(2,2,0)$			Example 1.9: $\xi=(2,2,1)$			Example 1.10: $\xi=(2,2,2)$		
a	N	S	L	N	S	L	N	S	L	N	S	L	N	S	L
ζ_a	0.20	0.25	0.30	0.05	0.20	0.25	0.05	0.10	0.45	0.05	0.10	0.25	0.05	0.075	0.10
$p_{t,1}$	2.04	2.04	2.04	2.35	2.1	2.1	2.06	2.06	1.06	2.05	2.05	1.3	2.5	2.5	2.5
$p_{t,2}$	1.54	1.54	1.54	2.54	1.54	1.54	1.9971	1.9971	0.99714	1.9	1.9	0.9	2.5	2.5	2.5
$U(\sigma\|a)$	8.8771	8.8109	8.7448	10.174	9.8246	9.7404	9.4397	9.2677	8.6118	9.2951	9.1042	8.5417	11.251	11.127	11.002
$V(\sigma\|a)$	1.8	1.775	1.75	0.75	0.95	0.9375	2.2	2.2	2.75	2.25	2.25	2.8125	0	0	0
Myopic-U	8.6979	8.6328	8.5677	10.961	10.584	10.475	11.539	11.346	10.731	11.429	11.215	10.694	11.324	11.206	11.089
Myopic-V	1.9792	1.9531	1.9271	0.10567	0.31861	0.31462	0.10449	0.10004	0.63359	0.10669	0.10215	0.64694	0	0	0

Note: For all 10 examples $\theta=1.5$, $\delta=0.8$, and $k=(\zeta_L-\zeta_S)/(\zeta_L-\zeta_N)$. If $p_{t,2}$ is accepted, the actual trading price is $kp_{t,2}$. All parameter values are given precisely. All prices and continuation values are given to 5 significant digits.

Table 2
"Union" examples of weak Markov equilibria

	Example 2.1: $\xi = (2,0,0)$			Example 2.2: $\xi = (2,1,0)$			Example 2.3: $\xi = (2,2,0)$		
a	N	S	L	N	S	L	N	S	L
ζ_a	0.20	0.60	0.80	0.20	0.40	0.80	0.20	0.25	0.80
$p_{t,1}$	2.35	1.35	1.35	2.35	1.60	1.35	2.35	2.35	1.35
$p_{t,2}$	2.70	1.70	1.70	2.75	1.75	1.75	2.30	2.30	1.30
$U(\sigma\|a)$	6.5943	6.2957	6.2957	6.6441	6.3927	6.3331	6.5051	6.3212	6.2288
$V(\sigma\|a)$	0.60	1.0	0.80	0.60	1.05	0.80	0.60	0.60	0.80
Myopic-U	6.9091	6.6818	6.6818	6.9091	6.6818	6.6818	6.9371	6.7457	6.7029
Myopic-V	0.34909	0.66182	0.46182	0.48	0.96	0.56	0.10971	0.11657	0.28229

Note: For all three examples $\theta = 1.5$, $\delta = 0.75$, and $k = 0.75$. If $p_{t,2}$ is accepted, the actual trading price is $kp_{t,2}$. All parameter values are given precisely. All prices and continuation values are given to 5 significant digits.

used in the examples vary greatly. When applying the model to strikes, the ζ's would depend on the particular industry or union–firm pair being considered. In an industry in which a clientele base is extremely important, the damage following a long strike may be severe so that ζ_L should be chosen close to 1. In another, one can imagine the expected future surplus is high regardless of current strike activity so that ζ_L should be close to 0. Listed below each possible state in the examples is ζ_a. The optimal price demands are given next, with the seller's (U) and buyer's (V) continuation values reported below the prices. The final two rows, labeled 'myopic-U' and 'myopic-V', report the lifetime discounted expected payoffs if the buyer and seller are myopic. The myopic payoffs are discounted by δ to make them make comparable to $U(\sigma|a)$ and $V(\sigma|a)$.

Table 2 gives three more examples with the parameters chosen to mimic a union–firm bargaining pair for which no strikes are particularly beneficial and long strikes are particularly damaging. In all three examples, $\theta = 1.5$, e.g., the marginal revenue product of labor is \$20 per hour in the good state and is \$12 per hour in the bad state. Similarly, $\delta = 0.75$ for all three examples, which corresponds to an annual discount factor of just under 10 percent if the contract lasts 3 years. A low δ can also reflect management's preference for maximizing profits (and stock prices) sooner rather than later. Unlike in Table 1, the three examples of Table 2 all set $k = 0.75$.[15] Following an immediate agreement, the bad state comes about, on average, just 1 in every 5 years as $\zeta_N = 0.20$. Following a long strike, the bad state comes about, on average, 4 of every 5 years as $\zeta_L = 0.80$. The examples in Table 2 all have the seller screening twice following an immediate agreement and pooling following a long strike. The seller's optimal strategy, however, varies with ζ_S. In Example 2.1, the bad state arises in 3 of every 5 years following a short strike, and the seller optimally pools following a short

[15] It is not necessary to think of this as indicating one-fourth of production time is lost during a short strike. Rather, payoffs are normalized to 0 following a long strike. Empirically, 97 percent of all strikes are settled within 6 months (Lemke, 1998). Thus, an appropriate interpretation is that a long strike lasts 6 months, and a short strike lasts one-fourth of that, or 1.5 months.

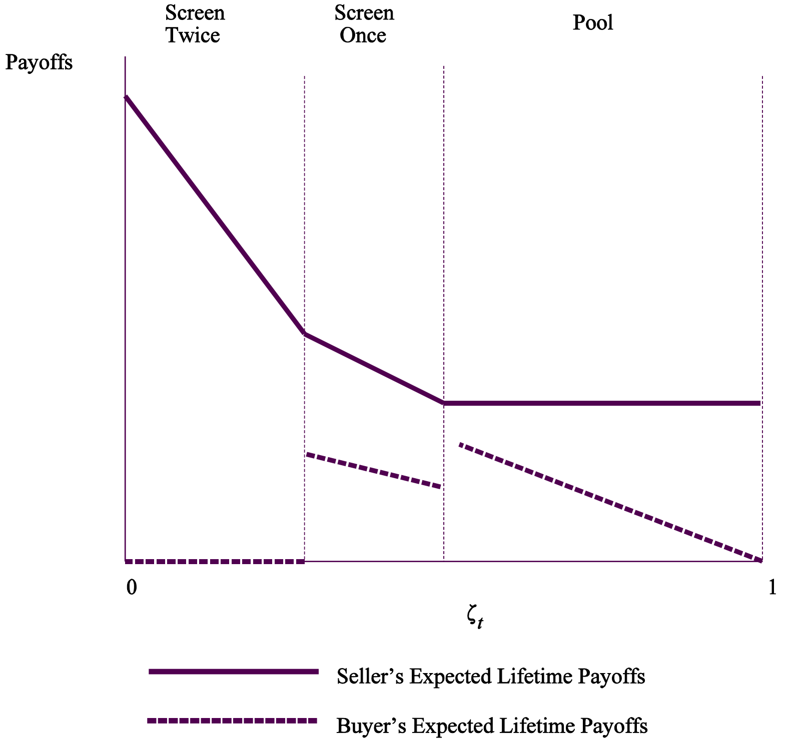

Fig. 1. Expected lifetime payoffs.

strike. In Example 2.2, the bad state arises, on average, 2 of every 5 years following a short strike, and the seller optimally screens once in such cases. And in Example 2.3, the bad state occurs, on average, just 1 in every 4 years following a short strike, and the seller's optimal strategy is to screen twice following a short strike.

As the seller moves first during each round of negotiations and chooses to pool, screen once, or screen twice in order to maximize her expected lifetime payoffs, the seller's lifetime expected payoff (weakly) decreases as the expected surplus decreases. Thus, the seller's lifetime expected payoffs are greatest following no strike and lowest following a long strike. This is restated in Theorem 2.

Theorem 2. *If* (σ^*, μ^*) *is a WME, then* $0 \leqslant U(\sigma^*|L) \leqslant U(\sigma^*|S) \leqslant U(\sigma^*|N)$.

Proof. See Appendix C. □

The buyer's continuation values, however, are not monotonic in ζ_t. If ζ_t is low, the seller screens with every demand leaving the buyer with no immediate surplus. If ζ_t is high, the seller pools leaving the buyer with some chance of receiving a positive payoff. Fig. 1 illustrates how the buyer's and seller's expected payoffs depend on the probability of receiving a low valuation. As the seller chooses the sequence of price demands, her expected surplus is weakly decreasing in ζ_t. However, the buyer's

Table 3
Example of efficiency gains from dynamic versus myopic bargaining

	Seller's dynamic strategy: $\xi = (0, 0, 0)$			Seller's myopic strategy: $\xi = (1, 0, 0)$		
a	N	S	L	N	S	L
ζ_a	0.35	0.75	0.90	0.35	0.75	0.90
$p_{t,1}$	1.94	1.94	1.94	2.2273	1.5	1.5
$p_{t,2}$	1.94	1.94	1.94	1.5	1.5	1.5
$U(\sigma\|a)$	9.7	9.7	9.7	7.8551	7.7841	7.7841
$V(\sigma\|a)$	1.05	0.65	0.50	0.9659	1.0227	0.87273
$J(\sigma\|a)$	10.75	10.35	10.20	8.821	8.8068	1.86568

Note: $\theta = 1.5$ and $\delta = 0.8$, and $k = 3/11$.

expected payoff increases as ζ_t decreases only if the decrease in ζ_t does not prompt the seller to adopt a more aggressive pricing strategy. That is, within the 'screen once' and 'pool' regions of Fig. 1, the buyer's expected payoff strictly increases as ζ_t decreases. When ζ_t decreases enough to entice the seller to play more aggressively, however, the buyer's expected payoff experiences a discrete jump downward.

The final issue of this section concerns whether the buyer and seller prefer to negotiate while understanding the dynamic nature of the game or if they would prefer to negotiate myopically, i.e., as if ζ_t came about exogenously at the start of period t. On one hand, understanding the dynamics allows the opportunity to carefully choose actions in order to maintain a high future expected joint surplus. On the other hand, knowing the dynamics may entice one or both parties to negotiate more aggressively to capture the potentially increased surplus. Depending on the parameter values, either of these forces may dominate.

Result 1. *Neither the seller nor buyer are predisposed to fare better or worse under the dynamic game compared to myopic bargaining.*

In Examples 1.1, 1.2 and 1.6, the seller prefers the dynamic game while the buyer prefers the myopic game. In the remaining examples of Table 1 and the examples in Table 2, these roles are reversed (with the buyer being indifferent in Example 1.10). In all of these examples, the parameter values are such that the seller plays according to the same ξ in the myopic game as she does in the dynamic game, although the price demands and randomization probabilities are different.

Table 3 gives an example in which the seller's pricing plan is different between the games—the seller screens once following an immediate agreement and pools otherwise in the myopic game, whereas she always pools in the dynamic game. By playing less aggressively in the dynamic game, the seller produces a higher expected joint surplus, and both agents benefit from this more cautious behavior.

6. Optimal price paths

In the static version of the game presented above, the seller lowers her price with each rejection, and consequently the sequence of price acceptances also decreases with strike duration. In a union–firm context, the standard prediction that wage agreements decrease with time is referred to as the union's downward sloping resistance curve. Using wage settlement data, Card (1990), McConnell (1989), Riddell (1980) and Vroman (1984) investigate the relationship between wages and strike duration and, with the exception of McConnell, find little evidence that wage agreements decline with strike duration.

The theoretical prediction that wages are decreasing with strike duration, however, depends strongly on the bargaining protocol. If the seller had the private information and the buyer made price offers, the sequence of accepted prices would increase with strike duration. Similarly, Fudenberg and Tirole (1983) and Cramton (1984) present models of two-sided private information in which price *demands* can be increasing with strike duration but *accepted* prices must, on average, be decreasing with strike duration.

The dynamic model presented here is novel in that price demands and average price acceptances can increase with strike duration. Examples 1.3, 1.5, 1.7, 2.1 and 2.2 show that under some WME the seller's second price demand will be higher than her first. For instance, in Example 2.2, following an immediate agreement the seller initially demands 2.35, but following a rejection then demands 2.75. In this same example, the seller screens once following a short strike by demanding 1.60 and, if that is rejected, countering with a price of 1.75. Recall, however, that at the time of the second demand, only k portion of the good remains so that the actual trading price is $kp_{t,2}$. Thus, prices are flow prices, similar to wages. It is easy to show under general conditions that actual trading prices in fact do decrease from the first to the second demand.

Furthermore, unlike the predictions of the two-sided bargaining games of Fudenberg and Tirole (1983) and Cramton (1984), average price acceptances can be negatively related to the current expected surplus (i.e., positively related to previous strike duration). In Example 2.1, this is readily obvious as the only time a second price demand is made and accepted is following an immediate agreement, and thus the accepted price is 2.7. The average of all prices when the first demand is accepted is trivially below 2.3, the highest first price demand. Example 2.2 is also associated with increasing average price acceptances. In this example, the probability that a high valuation buyer rejects an initial high price demand following an immediate agreement, ρ_N, is approximately 0.4524 (not reported in Table 2). Using ρ_N and the values of the ζs, the steady-state transition matrix can be easily solved:

From	To		
	No strike	Short strike	Long strike
No strike	0.438	0.362	0.200
Short strike	0.600	0.400	0.000
Long strike	1.000	0.000	0.000

Using these transition probabilities, the steady-state distribution of previous strike durations is such that 55.5 percent of current negotiations had no strike last time, 33.4 percent had a short strike, and 11.1 percent had a long strike. Weighting the seller's demands accordingly reveals that the average accepted price when no strike occurs is 1.88 while the average accepted price following a short strike is 2.32.

In contrast to the static game's downward sloped resistance curve, the dynamic bargaining game is associated with an *intertemporal* downward sloped resistance curve—the seller plays more aggressively the more quickly the *previous* negotiation was settled. In other words, price demands/agreements are negatively (or at least not positively) related to *previous* strike duration. Theorem 3 restates this result.

Theorem 3. *The seller demands* (*weakly*) *higher prices*, *the more quickly the previous negotiation was settled.*

Proof. See Appendix C. □

Theorem 3 is readily seen in all of the examples in the tables. In Example 1.5, the first price demand following no strike is 2.14. This falls to 1.64 if there was a short strike last time, and falls to 1.14 if there was a long strike last time. Likewise, in Example 2.2, following an immediate agreement the first price demand is 2.35. Following a short strike, the first demand is 1.6. And following a long strike, the first demand is 1.35.

The final observation on prices concerns the range of equilibrium prices. In static games, the seller's price demands necessarily fall between the buyer's low and high valuation, i.e., in the interval $[\theta, \theta + 1]$. In the dynamic game, however, because the buyer and seller both negotiate with an eye toward the future, equilibrium price demands can fall outside of $[\theta, \theta + 1]$. Specifically, the seller may at times be able to exploit the buyer's desire for an immediate settlement while at other times she may need to buy-off the buyer's preference for delay. For example, the buyer prefers a current strike if an immediate acceptance causes the seller to screen twice next time but a short strike worsens prospects causing the seller to pool or screen once next time.[16] It is even possible for the seller to ask for a price that is below her own valuation (i.e., below 0). Table 4 gives an example. Although she would never offer such a deal in a static model, the seller can prefer to offer a price below her current valuation in the dynamic game in exchange for expecting to be able to extract a high payoff in the future. The interpretation in a labor context is that the union may be willing, in bad economic times, to accept a wage below the non-union or reservation wage in exchange for a high expected future wage.

Similarly, it is also possible for the buyer to prefer agreeing sooner rather than later. This happens whenever the agreement time affects next contract's expected surplus but not the seller's optimal price plan. In this case, the buyer might accept a price above

[16] This behavior also appears in Kennan (2001).

Table 4
Example of negative prices

Seller's strategy: $\xi = (2, 2, 0)$			
a	N	S	L
ζ_a	0.05	0.10	0.70
$p_{t,1}$	0.915	0.915	−0.085
$p_{t,2}$	0.89125	0.89125	−0.10875
$U(\sigma\|a)$	16.393	16.298	15.488
$V(\sigma\|a)$	5.7	5.7	6

Note: $\theta = 1.5$, $\delta = 0.8$, and $k = 12/13$.

his current valuation in order to give himself a favorable position next time. In order for the buyer to accept a price above his actual valuation, however, it must be that the seller screens at most once during the subsequent negotiations. Thus, because the future looks brightest following no strike, the buyer will never accept a screening price above his valuation if it is the first demand in the negotiation. Result 2 summarizes all of these claims.

Result 2. *Concerning the sequence of price demands on the equilibrium path,*

(a) P^*_{10} *and* P^*_{20} *can be less than* θ (*and even negative*).
(b) P^*_{21} *can be greater than* $\theta + 1$.
(c) P^*_{11} *and* $P^*_{12} \leqslant \theta + 1$.

Verification. *See Appendix C.*

Examples of the statements made in Result 2 can be seen throughout Tables 1 and 2. In Examples 1.3, 1.4, 1.5, 1.8 and 1.9, P^*_{10} and/or P^*_{20} are less than θ. Part (b) of Theorem 5 is shown in Example 1.3, 2.1 and 2.2. In each of these examples, the seller screens twice following an immediate agreement. If the buyer rejects the first price demand, the seller counters with a price greater than $\theta + 1 = 2.5$. Notice that the seller agrees to this higher price, even when the first demand was less than $\theta + 1$, because the seller only screens once following a short strike, and this leaves the buyer the possibility of receiving a positive surplus during the next contract negotiation.

It is important to note that the results of this section do not rely on the two price—two valuation bargaining protocol, but rather they rely on the general discrete nature of the game. Specifically, some of the results require that a time come in the bargaining process when the seller makes a demand and, if the buyer rejects, the expected future surplus is then immediately, significantly, and irreversibly diminished. What price predictions would remain if the seller could continuously make demands and the future damage caused by strikes changed slowly and continuously is unclear. The model, therefore, best approximates real-world situations in which, even though bargaining takes place in continuous time, there are significant loses if the strike goes beyond a certain point in time. In September of 2002, for example, the labor agreement between the players' association and team owners for Major League Baseball was set to expire on the Friday before Labor Day. Moreover, the players' association had already called for a strike if a new agreement was not reached by then. Because a large portion of baseball's revenues come from television broadcasts of the post-season in October and because of the general consensus that the fan base would respond extremely negatively to not having the World Series played, there was a clear understanding on both sides that the failure to reach agreement by a certain date would have immediate, significant, and irreversible implications.

7. Strike incidence

Card (1988) showed that strike incidence for labor unions in the U.S. depends strongly on previous strike duration. Defining a short strike as lasting at most two weeks, the probability of striking during current negotiations is highest (roughly 40%) if previous negotiations resulted in a short strike and considerably lower (roughly 15% and 20%, respectively) if previous negotiations resulted in no strike or a long strike. Kennan (1995) and Lemke (1998) demonstrate a similar pattern for labor negotiations in Canada. Static or repeated games cannot explain this pattern as there is no interaction across contracts.

The dynamic game presented above is the first to be able to replicate the empirical pattern that strike incidence is higher following a short strike than following no strike or a long strike.[17] Consider the case when the seller always screens twice except following a long strike in which case she pools (as in Example 1.4). In this case, there are no strikes following a long strike, but there is a positive probability of encountering a strike following an immediate agreement or a short strike as the seller screens in these instances. When faced with a screening price, the buyer rejects and a strike arises only if his surplus value is low, but receiving a low valuation is more likely following a short strike than following an immediate agreement. Thus, strike incidence will be greater following a short strike than following no strike as the expected surplus value following a short strike is high enough to encourage screening on the seller's part but is not as high as the expected surplus following an immediate agreement. In Example 1.4, for instance, the probability of encountering a strike following no strike, a short strike, and a long strike is 0.30, 0.35, and 0 respectively. Likewise in example 1.8, the seller screens twice following no strike or a short strike. Thus, given the buyer's randomization probabilities ($\rho_N = 0.084$ and $\rho_S = 0.177$), the probability of striking following no strike, a short strike, and a long strike is 0.13, 0.26, and 0.0 respectively. Examples can also be generated in which there is a positive probability of striking following a long strike. By changing ζ_S to equal 0.12 in Example 1.9, the probability

[17] The dynamic games of Kennan (1995, 2001), Rustichini and Villamil (1996) and Vincent (1998) all assume only one offer can be made during each negotiation and are therefore unable to address issues of strike duration.

of striking following no strike, a short strike, and a long strike is 0.07, 0.29, and 0.25 respectively.

8. Concluding remarks

A dynamic bargaining model has been developed in which the agents' actions affect the conditions under which future negotiations take place. The set of equilibria studied captures a stationary or Markov structure. The equilibrium entails the seller making a certain number of screening demands followed (possibly) by a final pooling demand, where the number of screening demands made by the seller depends only on the outcome from the previous negotiation.

The predictions from the dynamic game are compared to those from a similar static model. It is shown that the equilibrium price path of the dynamic game is more flexible than that of the static game. In particular, whereas the range of equilibrium prices in the static game matches the range of possible buyer valuations, prices can range below and above this interval in the dynamic game. In some cases, prices can even fall below the seller's valuation. Also, whereas the seller optimally lowers her prices following each rejection by the buyer in the static game, the optimal sequence of price demands in the dynamic game is not necessarily monotonic as the seller either exploits the buyer's desire to settle or buys off the buyer's desire to postpone agreement. The dynamic setting is also able to produce the non-monotonic relationship between current strike incidence and previous strike duration discussed in the empirical union strikes literature.

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Appendix A. The myopic and static games

The myopic bargaining game is identical to the dynamic game put forth in the paper with a single exception—the agents negotiate without taking into account the relationship between current strike duration and the future expected surplus. Put differently, the agents negotiate as if $\delta = 0$. In the myopic game, consequently, each negotiation proceeds as if the agents are playing the static game (defined and solved below).

The static game is identical in protocol to negotiating a single contract in the dynamic game. The buyer's valuation is either θ or $\theta + 1$, and the seller makes at most two price demands. Once the buyer accepts a price or rejects the second one, payoffs are received and the game is over. As there is only one contract, let ζ be the probability that the buyer's valuation is low, and let p_1 and p_2 be the seller's first and second price demands.

Suppose p_1 is rejected. With her second demand the seller will either pool (and sell to both buyer types) or screen (and sell only if the buyer's valuation is high). Since the buyer receives a payoff of 0 following a second rejection, $p_2 = \theta$ is the pooling price and $p_2 = \theta + 1$ is the screening price. In deciding whether to pool or screen, the seller compares $k\theta$ to $k(1+\theta)(1-\zeta')$ where ζ' is the seller's updated belief that the buyer's type is low following an initial rejection. The seller's optimal strategy for p_2 is

$$p_2 = \begin{cases} \theta & \text{if } [1+\theta]^{-1} \leqslant \zeta', \\ 1+\theta & \text{otherwise.} \end{cases}$$

Now consider the seller's first price demand. At the start of negotiations, the seller can plan to pool immediately, screen once and then pool, or screen twice. Clearly the pooling price is θ and the twice screening price is $\theta + 1$. If the seller wants to screen once and then pool, the first price demand must leave a high valuation buyer with as much surplus as he would receive if he rejected and then accepted the second demand at the pooling price. That is, when planning to screen once, p_1 must satisfy $(\theta + 1 - p_1) \geqslant k(\theta + 1 - \theta)$, i.e., $p_1 = \theta + 1 - k$.

When the seller screens twice, a high valuation buyer randomizes between accepting and rejecting the first (of two) screens with probability ρ where ρ is the probability of rejecting. By randomizing, the buyer makes the seller indifferent between pooling and screening with her second demand (and is assumed to screen). Therefore, taking ρ as given, when $p_1 = \theta + 1$ is rejected, $\zeta' = \zeta/[\zeta + (1-\zeta)\rho]$. In order for ρ to make the seller indifferent between pooling and screening, $k\theta = k(\theta+1)(1-\zeta')$ is required. Substituting in for ζ' and solving for ρ yields, $\rho = \zeta\theta/(1-\zeta)$. Therefore, the seller's expected payoffs from each price sequence at the start of negotiations are

$$\theta \quad \text{if she pools immediately,}$$

$$(1-\zeta)(\theta + 1 - k) + k\zeta\theta \quad \text{if she screens once,}$$

$$(1-\zeta)(1-\rho)(\theta+1) + k\rho(1-\zeta)(1+\theta) \quad \text{if she screens twice.}$$

Comparing these three payoffs and defining $\zeta^1 = k([1+\theta][\theta - k\theta + k])^{-1}$ and $\zeta^2 = [1+\theta]^{-1}$, the seller's optimal strategy, the seller's expected payoffs, and the buyer's expected payoffs are

Parameter restriction	Seller's strategy	Seller's expected payoffs	Buyer's expected payoffs
$\zeta \in [0, \zeta^1]$	Screen twice	$\theta + 1 - \zeta(\theta+1)(\theta + 1 - k\theta)$	0
$\zeta \in [\zeta^1, \zeta^2]$	Screen once	$\theta + 1 - k - \zeta(\theta+1)(1-k)$	$k(1-\zeta)$
$\zeta \in [\zeta^2, 1]$	Pool	θ	$1-\zeta$

Notice that the seller's expected payoff increases as ζ decreases. The buyer's expected payoff, however, increases only as ζ decreases within $[\zeta^1, \zeta^2]$ or within $[\zeta^2, 1]$. His payoff falls as ζ crosses ζ^1 or ζ^2 from above.

Appendix B. Tight prices

Let $V^{\mathrm{L}}(\sigma|a)$ and $V^{\mathrm{H}}(\sigma|a)$ be the continuation value in state a to a low and high valuation buyer respectively when the strategy profile is σ. Because tight prices extract all of the low valuation surplus or are so high that a low valuation buyer finds it best to reject the demand and suffer a long strike $V^{\mathrm{L}}(\sigma|a) = \delta V(\sigma|L)\ \forall a \in \{N, S, L\}$.

Pooling ($\xi_a = 0$): When the seller pools in state a,

$$V^{\mathrm{L}}(\sigma|a) = \theta - P^*_{10} + \delta V(\sigma|N) = \delta V(\sigma|L),$$

where the first equality follows from the expected lifetime payoff to the low valuation buyer from accepting the pooling demand and the second equality follows from above. (The equilibrium prices depend on σ, but for the moment, this dependency is omitted from the notation.) Thus,

$$P^*_{10} = \theta + \delta[V(\sigma|N) - V(\sigma|L)].$$

Further, because a high valuation buyer will also accept the pooling demand,

$$V^{\mathrm{H}}(\sigma|a) = \theta + 1 - P^*_{10} + \delta V(\sigma|N) = 1 + \delta V(\sigma|L).$$

Thus, when $\xi_a = 0$,

$$V(\sigma|a) = \zeta_a V^{\mathrm{L}}(\sigma|a) + (1 - \zeta_a) V^{\mathrm{H}}(\sigma|a) = 1 - \zeta_a + \delta V(\sigma|L).$$

Screening once ($\xi_a = 1$): When the seller screens once in state a,

$$V^{\mathrm{L}}(\sigma|a) = k[\theta - P^*_{20}] + \delta V(\sigma|S) = \delta V(\sigma|L),$$

because a low valuation buyer will reject the first price demand and accept the second. Thus,

$$P^*_{20} = \theta + (\delta/k)[V(\sigma|S) - V(\sigma|L)].$$

In equilibrium, a high valuation buyer will be indifferent between accepting and rejecting the initial screening demand because prices are tight. Therefore,

$$V^{\mathrm{H}}(\sigma|a) = \theta + 1 - P^*_{11} + \delta V(\sigma|N) = k[\theta + 1 - P^*_{20}] + \delta V(\sigma|S).$$

Substituting in for P^*_{20} yields

$$P^*_{11} = \theta + 1 - k + \delta[V(\sigma|N) - V(\sigma|L)].$$

Thus, when $\xi_a = 1$,

$$V(\sigma|a) = \zeta_a V^{\mathrm{L}}(\sigma|a) + (1 - \zeta_a) V^{\mathrm{H}}(\sigma|a) = (1 - \zeta_a)k + \delta V(\sigma|L).$$

Screening twice ($\xi_a = 2$): When the seller screens twice in state a, both price demands must extract all of the surplus from a high valuation buyer. Thus,

$$V^{\mathrm{L}}(\sigma|a) = V^{\mathrm{H}}(\sigma|a) = \delta V(\sigma|L).$$

Moreover, because a high valuation buyer is indifferent between accepting P^*_{12}, accepting P^*_{21}, and enduring a long strike, P^*_{12} and P^*_{21} must satisfy

$$\theta + 1 - P^*_{12} + \delta V(\sigma|N) = k[\theta + 1 - P^*_{21}] + \delta V(\sigma|S) = \delta V(\sigma|L).$$

Thus, when $\xi_a = 2$,

$$P^*_{12} = \theta + 1 + \delta[V(\sigma|N) - V(\sigma|L)],$$

and

$$P^*_{21} = \theta + 1 + (\delta/k)[V(\sigma|S) - V(\sigma|L)].$$

Appendix C. Proofs

Claim 1. *When restricted to weak Markov strategies, the seller pools, screens once, or screens twice on the equilibrium path.*

Proof. Let $\sigma^{\mathrm{B}} \in \Sigma^{\mathrm{B}}$ and $\sigma^{\mathrm{S}} \in \Sigma^{\mathrm{S}}$, and define $\sigma = (\sigma^{\mathrm{B}}, \sigma^{\mathrm{S}})$. Denote by $V(\sigma|a)$, or, more simply, $V(a)$, the buyer's expected lifetime discounted payoffs when negotiating a contract when the current state is $a \in \{N, S, L\}$. Thus, a buyer receives a lifetime expected payoff of $\delta V(L)$ upon rejecting both price demands, where δ is the discount rate.

Suppose the buyer rejected the first demand and is faced with accepting or rejecting the seller's second demand. The buyer accepts if $k[\theta + n_t - p_{t,2}] + \delta V(S) \geqslant \delta V(L)$. Thus, a low valuation buyer accepts if $p_{t,2} \leqslant p_{20}$ where

$$p_{20} = \theta + (\delta/k)[V(S) - V(L)],$$

while a high valuation buyer accepts if $p_{t,2} \leqslant p_{21}$ where

$$p_{21} = \theta + 1 + (\delta/k)[V(S) - V(L)].$$

(Note that p_{20} and p_{21} are the pooling and screening prices respectively.) As a low (high) valuation buyer accepts any price up to the pooling (screening) price, the seller must demand p_{20} or p_{21} in the second period to maximize her expected payoff. Further, when $p_{t,2} = p_{20}$, a low valuation buyer receives a payoff of $\delta V(L)$, which is what he receives if he rejects the demand, while a high valuation buyer receives a payoff of $k + \delta V(L)$. When $p_{t,2} = p_{21}$, both buyer types receive a payoff of $\delta V(L)$—the low valuation buyer because he rejects the demand and a high valuation buyer because the screening price extracts as much surplus as possible from him.

Now consider the first demand. A low valuation buyer receives $\delta V(L)$ if he rejects $p_{t,1}$, and therefore accepts any price that leaves him with at least this much surplus. Thus, a low valuation buyer accepts $p_{t,1}$ if $p_{t,1} \leqslant p_{10}$ where

$$p_{10} = \theta + \delta[V(N) - V(L)].$$

Following a rejection, a high valuation buyer will receive either $k+\delta V(L)$ if the seller pools or $\delta V(L)$ if the seller screens with her second demand. Thus, a high valuation buyer accepts $p_{t,1}$ if $p_{t,1} \leqslant p_{11}$ where

$$p_{11} = \theta + 1 - k + \delta[V(N) - V(L)]$$

and he believes the seller will pool with her second demand or if $p_{t,1} \leqslant p_{12}$ where

$$p_{12} = \theta + 1 + \delta[V(N) - V(L)]$$

and he believes the seller will screen with her second demand. Thus, the seller either chooses $p_{t,1} = p_{10}$ and sells to both buyer types or chooses $p_{t,1} \in [p_{11}, p_{12}]$ so that a low valuation buyer rejects but a high valuation buyer accepts or rejects or randomizes based on his belief of what the seller will do following a rejection. The proof will be complete, therefore, with the seller pooling at a price of p_{10}, screening once with a price of p_{11} followed by p_{20}, or screening twice with a price of p_{12} followed by p_{21}, if it can be shown that $p_{t,1} \in (p_{11}, p_{12})$ cannot be supported on the equilibrium path.

Suppose $p_{t,1} \in (p_{11}, p_{12})$ is rejected. The seller then updates her belief of the buyer's type. Suppose the seller's updated belief leads her to screen with her second demand. (Recall that if the seller is indifferent between pooling and screening, she is assumed to screen.) In this case, all high valuation buyers should have accepted the initial demand. But if all high valuation buyer's accept the first demand, then following a rejection the seller should pool with her second demand. Likewise, suppose the seller's updated belief leads her to pool with her second demand. In this case, all high valuation buyers should have rejected the initial demand. But if all high valuation buyers reject the first demand, then following a rejection the seller should screen with her second demand given that she found it best to screen with her first demand. In either case, therefore, $p_{t,1} \in (p_{11}, p_{12})$ cannot be supported on the equilibrium path. The only screening possibilities, therefore, are for $p_{t,1} = p_{11}$ which high valuation buyers accept (and is followed by a pooling demand if rejected) or for $p_{t,1} = p_{12}$ which only some high valuation buyers accept and is followed by another screening demand if rejected. $\square$

Theorem 1. *For any $\omega \in \Omega$, if (σ^*, μ^*) is a WME associated with a pricing strategy of ξ, then $\xi \in \Xi^*$ where*

$$\Xi^* = \{(\xi_N, \xi_S, \xi_L)\colon\ \xi_N, \xi_S, \xi_L \in \{0,1,2\} \text{ and } \xi_N \geqslant \xi_S \geqslant \xi_L\}.$$

Proof. Because the optimal prices depend on the buyer's continuation values, showing Theorem 1 is tedious and is done by showing that each ξ which is in Ξ but is not in Ξ^* can never produce a weak Markov equilibrium. A proof for $\xi = (0,0,1)$ not being in Ξ^* is offered below to illustrate the style of the proof. A proof for $\xi = (1,1,2)$ not being in Ξ^* is also offered to demonstrate how cases when the seller screens twice can be shown.

Suppose $\xi = (0,0,1)$ is associated with a WME. Because the seller's optimal choice is to pool following an immediate agreement and to screen once following a long strike, we have that

$$U(N) = P_{10}^* + \delta U(N) \geqslant (1-\zeta_N)[P_{11}^* + \delta U(N)] + \zeta_N[kP_{20}^* + \delta U(S)]$$

and

$$U(L) = (1-\zeta_L)[P_{11}^* + \delta U(N)] + \zeta_L[kP_{20}^* + \delta U(S)] \geqslant P_{10}^* + \delta U(N).$$

Thus, for $\xi = (0,0,1)$ to be associated with an equilibrium, it must be that

$$(\zeta_L - \zeta_N)[P_{11}^* + \delta U(N) - kP_{20}^* - \delta U(S)] \leqslant 0. \tag{C.1}$$

Solving Eqs. (6), (7), and (9) simultaneously yields

$$P_{10}^* = \theta + \delta[1 - k + k\zeta_L - \zeta_N], \quad P_{20}^* = \theta + (\delta/k)[1 - k + k\zeta_L - \zeta_S],$$

and

$$U(N) = U(S) = P_{10}^*/(1-\delta).$$

Substituting into (C.1) yields

$$(\zeta_L - \zeta_N)[(\theta+1)(1-k) + \delta(\zeta_S - \zeta_N)] \leqslant 0$$

which is a contradiction as all bracketed terms are positive. Thus, $\xi = (0,0,1)$ cannot be associated with a WME.

Suppose instead that $\xi = (1,1,2)$ is associated with a WME. Eqs. (6) and (7) then yield

$$V(N) = k(1-\zeta_N), \quad V(S) = k(1-\zeta_S), \quad V(L) = 0,$$

$$P_{10}^* = \theta + \delta k(1-\zeta_N), \quad \text{and} \quad P_{20}^* = \theta + \delta(1-\zeta_S).$$

Notice that $P_{11}^* - kP_{20}^* = (1-k)(1+\theta) + \delta k(\zeta_S - \zeta_N) \geqslant 0$. Using Eq. (9), $U(N)$ and $U(S)$ can be determined, but of importance here is that

$$U(N) - U(S) = \frac{(\zeta_S - \zeta_N)(P_{11}^* - kP_{20}^*)}{1 - \delta(\zeta_S - \zeta_N)} \geqslant 0.$$

Because the seller screens twice following a long strike, Eq. (11) plays a central role in the proof. Let $K = \theta + (\delta/k)[J(S) - J(L)]$ so that $\rho_N = K\zeta_N/(1-\zeta_N)$ and $\rho_L = K\zeta_L/(1-\zeta_L)$. (If $\rho_L > 1$, the proof follows similarly and is slightly easier by substituting 1 for ρ_L.) Now, because the seller screens once following an immediate agreement but screens twice following a long strike, it follows that

$$(1-\zeta_N)[P_{11}^* + \delta U(N)] + \zeta_N[kP_{20}^* + \delta U(S)]$$
$$\geqslant (1-\zeta_N)(1-\rho_N)[P_{12}^* + \delta U(N)] + [(1-\zeta_N)\rho_N + \zeta_N][kP_{20}^* + \delta U(S)]$$

and

$$(1-\zeta_L)[P_{11}^* + \delta U(N)] + \zeta_L[kP_{20}^* + \delta U(S)]$$
$$\leqslant (1-\zeta_L)(1-\rho_L)[P_{12}^* + \delta U(N)] + [(1-\zeta_L)\rho_L + \zeta_L][kP_{20}^* + \delta U(S)],$$

where the last part of each equation comes from knowing that the seller is indifferent between screening and pooling with her second demand. On the equilibrium path, she chooses to screen, but the expected payoffs from screening for a second time or pooling are the same. Subtracting the bottom from the top and simplifying yields

$$(\zeta_L - \zeta_N)[k + K(P_{12}^* - kP_{20}^* + \delta[U(N) - U(S)])] \leqslant 0$$

which is a contradiction as $(\zeta_L - \zeta_N) > 0$, $K > 0$, $P^*_{12} - kP^*_{20} > 0$, and $U(N) - U(S) > 0$. □

Theorem 2. *If* (σ^*, μ^*) *is a WME, then* $0 \leqslant U(\sigma^*|L) \leqslant U(\sigma^*|S) \leqslant U(\sigma^*|N)$.

Proof. First note that $U(\sigma^*|a) \geqslant 0$ for all $a \in \{N, S, L\}$, otherwise the seller could increase her lifetime payoff to at least zero by demanding prohibitively high prices. Let ξ^* be the seller's pricing plan associated with σ^*. Now, suppose $\xi^*_N = \xi^*_S = 1$ so that

$$U(a) = (1 - \zeta_a)[P^*_{11} + \delta U(N)] + \zeta_a[kP^*_{20} + \delta U(S)] \quad \text{for } a \in \{N, S\}.$$

Therefore,

$$U(N) - U(S) = (\zeta_S - \zeta_N)[P^*_{11} + \delta U(N) - kP^*_{20} - \delta U(S)]$$

as $\zeta_N \leqslant \zeta_S$ and $P^*_{11} - kP^*_{20} \geqslant 0$ as shown above. Similar exercises can be carried out to show $U(N) \geqslant U(S)$ when $\xi^*_N = \xi^*_S = 2$ and when $\xi^*_N = \xi^*_S = 0$.

Next, consider the case when $\xi^*_N = 1$ and $\xi^*_S = 0$. In this case $U(S) = P^*_{10} + \delta U(N)$, and since the seller could pool following no strike but chooses not to, it must be that $U(N) \geqslant P^*_{10} + \delta U(N)$. Thus, $U(N) \geqslant U(S)$. Similarly, $U(N) \geqslant U(S)$ if $\xi^*_N = 2$ and $\xi^*_S = 0$ or if $\xi^*_N = 2$ and $\xi^*_S = 1$.

This covers all of the possible relationships between ξ^*_N and ξ^*_S, and we have $U(S) \leqslant U(N)$. The above arguments depend only on the seller's possible deviation actions and that $\zeta_N \leqslant \zeta_S$. Since the set of possible deviations by the seller is the same regardless of which σ is used to generate the equilibrium and because $\zeta_S \leqslant \zeta_L$, it follows that $U(L) \leqslant U(S)$. □

Theorem 3. *The seller demands (weakly) higher prices, the more quickly the previous negotiation was settled.*

Proof. Recall from Theorem 1 that each ξ in Ξ^* is such that $\xi_N \geqslant \xi_S \geqslant \xi_L$, and from Eq. (6), $P^*_{10} \leqslant P^*_{11} \leqslant P^*_{12}$ and $P^*_{20} \leqslant P^*_{21}$. Consider two buyer–seller pairs, labeled α and β, that settle contract t simultaneously, and suppose the α contract is associated with a larger agreed upon price. Thus, the α-seller must have been playing tougher when both buyers accepted the current price demand. The α-seller plays tougher than the β-seller, however, if and only if ξ^* dictates as much, which happens if and only if the α pair settled more quickly during the negotiations over contract $t-1$. □

Result 2. *Concerning the sequence of price demands on the equilibrium path,*

(a) P^*_{10} *and* P^*_{20} *can be less than* θ *(and even negative).*
(b) P^*_{21} *can be greater than* $\theta + 1$.
(c) P^*_{11} *and* $P_{12*} \leqslant \theta + 1$.

Verification. Examples of parts (a) and (b) are given in Tables 1 and 2. All that remains is part (c). If $\xi_N = 0$, then $\xi^* = (0, 0, 0)$ and neither P^*_{11} nor P^*_{12} are on the equilibrium path. If $\xi_N = 1$, so that the seller screens once following an immediate agreement, then $\xi^* \in \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$. In each case, Eq. (7) can be used to figure $V(N)$ and

$V(L)$. When $\xi^* = (1,0,0)$ and $\xi^* = (1,1,0)$, $P^*_{11} = \theta + 1 - k(1-\delta) - \delta(1 + k\zeta_N - \zeta_L) \leqslant \theta + 1$. When $\xi^* = (1,1,1)$, $P^*_{11} = \theta + 1 - k[1 - \delta(\zeta_L - \zeta_N)] \leqslant \theta + 1$. If $\xi_N = 2$, so that the seller screens twice following an immediate agreement, then by Eq. (7) we have that $V(N) = \delta V(L)$, which implies that $V(N) - V(L) = -(1-\delta)V(L) \leqslant 0$. Thus, by Eq. (7), $P^*_{12} \leqslant \theta + 1$ (as must be P^*_{11}).

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