Extending Monopoly Power Under Joint Production

Robert J. Lemke
Department of Economics
Lake Forest College
lemke@lakeforest.edu
847-735-5143

Kristina M. Lybecker
Department of Economics
The Colorado College
lybecker@lakeforest.edu
719-632-2966

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Abstract

This article provides an exercise on monopoly power under joint production for advanced undergraduate or graduate students. The analysis is motivated by an example of joint production from the markets for blood products. Joint production is used to illustrate questions surrounding the leveraging of monopoly power. Specifically, could a firm with a monopoly over one jointly produced good extend market power to the non-monopolized good? The analysis considers the potential for a dual monopoly, limit pricing, and a shared market in which the firms compete ala Cournot. The exercise is intended as an in-class exercise, problem set, or take-home exam.

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1. Introduction

Economists have developed a significant literature concerning whether a monopolist can gain additional rents by leveraging its monopoly power to foreclose competition in a second, complementary market through tying the sale of its monopoly commodity to the sale of the commodity in the second market. The Chicago School’s refutation of leveraging, the “one monopoly rent theorem,” may be traced to Director & Levi (1956). Director provided a simple proof to establish that leveraging cannot generate additional monopoly profits. This claim, however, has been challenged more recently. Specifically, Kaplow (1985) identified several heroic assumptions necessary to support the leverage hypothesis, including the reliance on a static framework and the assumption of perfect markets. Choi (1996, 2004) moved away from a static framework to highlight the application of leverage through the interaction of tying and R&D incentives. Whinston (1990) relaxed the perfect market assumption (allowing for increasing returns) to demonstrate that leveraging through tying may be profitable when a monopolist can alter the structure of the second market. Nalebuff (2004) established that Director’s result depends on the consumption of the goods in fixed proportions and once this assumption is replaced with consumption in variable proportions, monopoly power can be extended. Brennan and Kimmel (1986) also departed from the standard market assumptions in an attempt to show that a monopolist may leverage more rents by tying to jointly produced commodities.

Following the work of Brooks and Lybecker (2007) and Brennan and Kimmel (1986), the analysis in this paper, which is motivated with an example from the market for blood and blood products, focuses on the monopolization of goods tied at the point of production. Specifically, this exercise examines whether, and under what conditions, a monopolist over one jointly produced good would seek to extend the monopoly to the other jointly produced good. The analysis considers, in turn, the potential for a dual monopoly, a market shared by firms competing a la Cournot, as well as limit pricing. We also examine issues regarding efficiency.

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1 For a discussion of the history of leverage theory and a review of the literature, see Whinston (1990) and Posner and Easterbrook (1981).
2 This exercise presents a simplified version of the model and results presented in Brooks and Lybecker (2007).
We develop a simple model of joint production in which two firms compete ala Cournot competition in one market, but one of the firms is a monopolist in the second market. Demand is assumed to be identical across markets, and each firm faces the same constant marginal cost of production. In addition to holding a monopoly right in one market, the monopolist’s technology affords it benefits due to the joint production nature of the goods. In particular, the monopolist is able to spread the marginal cost of production across two markets, while the competing firm must recover all costs solely in the non-monopolized market. In the motivating example given in section 2, the monopolist faces a constant marginal cost for whole blood, which may be processed to generate both plasma and red blood cells – two goods with distinct and independent markets. Meanwhile, the remaining firm faces the same constant marginal cost for whole blood, but it can only process its whole blood for sale in the market for red blood cells due to the patent held by the monopolist in producing plasma.

We characterize the complete solution to the model in sections 3 – 7. The solution captures several interesting features of industrial organization. Under certain parameter values, the monopolist finds it best to not sell all of its output in one of the markets. Under other parameter values, the monopolist finds it best to engage in limit pricing in the second market (thus precluding the second firm from selling output in either market). There are still other ranges of parameter values under which the two firms compete as usual under Cournot.

The purpose of this article, in essence, is to provide a potential problem set for instructors of a class in industrial organization. The model we develop, which is rooted in the classic Cournot model, can be used to emphasize several fundamental concepts of industrial organization, including joint production, limit pricing, and extending monopoly power across markets. Moreover, the problem we present, while as simple as possible, is also fairly complex. The entire solution is probably best suited as an in-class exercise, a problem set, or a take-home exam for advanced undergraduate or graduate students. Section 2 describes an application of the problem using the markets for blood products. The intent is to provide instructors with a clear example of the model. Sections 3 then sets up the problem and sections 4 – 7 solve the complete problem in detail. In section 8 we consider the efficiency implications, especially in
2. The Joint Production of Blood Products – SD Plasma and Red Blood Cells

In May of 1998, the US Food and Drug Administration approved a technology for removing certain viruses, including HIV and hepatitis B and C, from plasma. The approval of solvent-detergent plasma (SD plasma) came in the midst of a US Department of Justice antitrust investigation of the American Red Cross, focused precisely on this technology and the exclusive contract the Red Cross acquired with manufacturer V.I. Technologies. Exclusive control of the technology raised concerns that some independent blood banks could be forced out of business. The DOJ argued that due to the joint production of blood products, the Red Cross would use the SD plasma monopoly to exploit other markets and secure a second monopoly.

At issue was whether the patent over SD plasma would provide the American Red Cross with a cost advantage and the ability to limit competition from the independent blood centers which provide half of the nation’s blood supply. The cost advantage stems from the joint-production nature of blood products. The extraction of a unit of whole blood results in the potential production of several blood products with distinct and independent markets: plasma, red blood cells, white blood cells, and platelets. While the patented technology clearly entitled the American Red Cross to a monopoly in the market for SD plasma, concern surrounded the potential to extend this monopoly to the highly profitable market for red blood cells.
Ultimately, the DOJ investigation was shelved when an alternative plasma-scrubbing technology was developed, eliminating the Red Cross monopoly. It is important to note, however, that the case was never closed and the potential for the same issue to arise in the future clearly still exists.\(^3\) For example, V.I. Technologies is currently in the clinical trial stages of testing a new technology that removes pathogens from red blood cells. Thus, the potential for the monopolization of the market for a single blood product remains.

The model that follows examines the markets for two jointly produced goods, such as SD plasma and red blood cells. In one market, two firms compete ala Cournot, an assumption that reasonably mirrors the competitive nature of the American Red Cross and Blood Centers of America (the independent blood banks) each of which has approximately half of the US market. Similarly, in order to capture the American Red Cross’s patented innovation for producing SD plasma, it is assumed that one firm acquires a patent that provides it a monopoly in the market for one of the jointly-produced goods.

3. The Model

Consider two goods, \(A\) and \(B\), linked in production such that the production of a single unit of good \(A\) also generates (costlessly) a unit of good \(B\) (and vice versa). Denote these two markets by superscripts \(A\) and \(B\). Denote two firms by subscripts 1 and 2. Firm 1 is a patent holder in market \(A\), precluding firm 2 from selling any output there. The two firms (potentially) engage in Cournot competition in market \(B\). It is further assumed that either firm can costlessly dispose of any of its product in either market. Noting that firm 2 is prohibited from selling in market \(A\), firm 2 will always costlessly dispose of the entire quantity of good \(A\) it produces.

Firm 1 chooses a production level, \(q_1\), which allows it to sell up to that quantity in both markets. Denote by \(q_1^A\) and \(q_1^B\) the amount of output firm 1 decides to sell in each market subject to the

\(^3\) The potential for the monopolization of SD plasma was sufficiently concerning that the Pentagon joined the complaint, likely because of the Pentagon’s need for blood during wartime. Because the case officially remains open, however, not much is known concerning the precise reasons why the Pentagon joined the suit.
constraints that \( q_1^A \leq q_1 \) and \( q_1^B \leq q_1 \). Total revenue for firm 1 is \( p^A q_1^A + p^B q_1^B \). Firm 1 faces no fixed costs and a constant marginal cost of production of \( c \) where \( c \geq 0 \) so that its total costs are \( cq_1 \).

Firm 2’s problem, though easier conceptually, can be explained in an identical fashion. Firm 2 chooses an output level, \( q_2 \). Concurrent with making its total production decision, firm 2 also decides how much to sell in the \( B \) market, denoted by \( q_2^B \), subject to the constraint that \( q_2^B \leq q_2 \). As firm 1 is a patent holder in market \( A \), firm 2 cannot sell any output in that market so that firm 2 costlessly disposes of all of its market \( A \) output (i.e., \( q_2^A = 0 \)). Total revenue for firm 2 is \( p^B q_2^B \). Firm 2 faces no fixed costs and a constant marginal cost of production of \( c \) where \( c \geq 0 \) so that firm 2’s total costs are \( cq_2 \).

Finally, let total quantity supplied to each market be denoted by \( Q^A \) and \( Q^B \) where

\[
Q^A = q_1^A + q_2^A = q_1^A \quad \text{and} \quad Q^B = q_1^B + q_2^B.
\]

Assume that demand in both markets is identical (and linear). In particular, market clearing prices are \( p^A = \alpha - Q^A \) and \( p^B = \alpha - Q^B \) where \( \alpha \geq c \).

It is assumed that either (or both) firms may costlessly dispose of units of either good that it elects not to sell. However, since firm 2 cannot sell in market \( A \) and it would never be profit maximizing for firm 2 to costlessly dispose in market \( B \), firm 2 always sets \( q_2^A = 0 \) and \( q_2^B = q_2 \). Thus, we will say that firm 2 never costlessly disposes in either market. In contrast, although firm 1 can sell in both markets, it will never costlessly dispose in both markets simultaneously. Therefore, after making its total production decision, firm 1 is said to costlessly dispose in market \( A \) if \( q_1^A < q_1^B = q_1 \) and is said to costlessly dispose in market \( B \) if \( q_1^B < q_1^A = q_1 \).

### 4. Dual Monopoly

We define firm 1 as being a dual monopolist whenever demand and cost conditions are such that firm 2 will never enter the \( B \) market even if firm 1 is not engaged in predatory behavior via limit pricing (addressed in section 7). Put differently, firm 1 is a dual monopolist whenever its reaction function in

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\(^4\) It is important to note that while the marginal costs of production faced by firms 1 and 2 are identical, they could easily differ, demonstrating that that main results stem from something other than a cost advantage.
market $B$ from the standard Cournot model results in no production by firm 2. If firm 1 is a dual monopolist, its profit function is

$$\pi_{1}^{\text{DM}} = p^A q_1^A + p^B q_1^B - cq_1$$

where $q_1 = \max\{q_1^A, q_1^B\}$.

In order to solve for the conditions under which firm 1 is a dual monopolist, one must first determine how firm 1 treats its costs. Since production is costly, firm 1 will never costlessly dispose in both markets. Without loss of generality, suppose that firm 1 is a dual monopolist and maximizes its profit by costlessly disposing in market $B$ so that $q_1^B < q_1^A = q_1$. Given identical demand in markets $A$ and $B$, firm 1’s marginal revenue in market $A$ is less than its marginal revenue in market $B$ as $q_1^A > q_1^B$. At the same time, the marginal cost of supplying more output to market $B$ is zero as firm 1 is costlessly disposing in market $B$. Taken together, these conditions contradict firm 1 maximizing its profits – either firm 1 should increase sales in market $B$ if the marginal revenue of doing so is positive (as the marginal cost of doing so is zero), or it should restrict sales (and overall production) in market $A$ if marginal revenue in $B$ equals zero as marginal revenue in market $A$ is then negative but the marginal cost is $c$.

Thus, when firm 1 is a dual monopolist, it will always sell the same amount of output in both markets.

As a dual monopolist, firm 1 chooses $q_1$ and sells the entire quantity in both markets $(q_1^A = q_1^B = q_1)$. Its profit are then:

$$\pi_{1}^{\text{DM}} = p^A q_1 + p^B q_1 - cq_1 = (p^A + p^B - c)q_1 = (2\alpha - 2q_1 - c)q_1.$$ 

Setting the first order condition equal to zero and solving for $q_1$ yields:

$$q_1 = q_1^A = q_1^B = \frac{2\alpha - c}{4} = \frac{\alpha - (c / 2)}{2} \quad \text{and} \quad p^A = p^B = \frac{2\alpha + c}{4}.$$
Notice that the right-most quantity expression reveals that the monopolist acts as if its marginal cost is $c/2$ in both markets.\(^5\)

Lastly, the demand and cost conditions that provide for firm 1 being a dual monopolist must be determined. The conditions for dual monopoly are simple: if the quantity that firm 1 would supply to market $B$ in the absence of competition from firm 2 is such that it drives the price in market $B$ to be at most $c$, then firm 1 will always act like a monopolist in market $B$ and firm 2 will never produce. Using the solution above when firm 1 is a monopolist in market $B$, $p_B = (2\alpha + c)/4$ is less than or equal to $c$ when $c \geq 2\alpha/3$. Thus, firm 1 is a dual monopoly whenever $c \geq 2\alpha/3$. Conversely, firms 1 and 2 compete in market $B$ whenever $c < 2\alpha/3$.

**The results under dual monopoly (DM) are as follows.** Dual monopoly occurs when $c \geq 2\alpha/3$. Equilibrium quantities, prices, and profits are:

$$q_1^A = q_1^B = \frac{2\alpha - c}{4}; \quad q_2^A = q_2^B = 0; \quad p^A = p^B = \frac{2\alpha + c}{4}; \quad \pi_1^{DM} = \frac{(2\alpha - c)^2}{8}; \quad \text{and} \quad \pi_2^{DM} = 0.$$ 

A summary of the results for each case may be found in Table 1.

5. **Costless Disposal by Firm 1**

Whenever $c < 2\alpha/3$, firm 1 will face competition in market $B$ from firm 2.\(^6\) The question arises, when (if ever) will firm 1 willingly engage in costless disposal in either market? We consider market $A$ and market $B$ each in turn.

In addition to never choosing to costlessly dispose in both markets simultaneously, firm 1 will never costlessly dispose in market $A$. Suppose that firm 1 maximizes profits by costlessly disposing in market $A$ so that $q_1^A < q_1^B = q_1$. Firm 1 will increase sales in market $A$ until marginal revenue in market $A$ equals zero (which requires $q_1^A = \alpha/2$) and costlessly dispose of its remaining output that it could

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\(^5\) This result is expected as the firm will want to treat each market identically which, in this case, requires the firm to split the marginal cost of production equally across markets. Any other treatment of marginal cost would lead the firm to over-produce (under-produce) in the market that it associates with the lower (higher) cost.

\(^6\) This assumes that firm 1 doesn’t engage in limit pricing in market $B$, a possibility that is treated in section 7.
otherwise sell in market $A$. Firm 1’s profit in market $B$, holding fixed firm 2’s output decision and recognizing that marginal cost in market $B$ is $c$ for firm 1, is $(\alpha - q_1^B - q_2^B - c) \cdot q_1^B$, which yields a best response function of $q_1^B = (\alpha - c - q_2^B) / 2$. Firm 1’s optimal amount of output to sell in the $B$ market, therefore, is less than $\alpha / 2$ as $c$ and $q_2^B$ are both non-negative. But this contradicts the assumption that $q_1^A < q_1^B$ as $q_1^A = \alpha / 2$ when firm 1 is costlessly disposing in market $A$. Thus, costlessly disposing in market $A$ cannot be profit maximizing for firm 1.

In contrast, firm 1 will find it optimal to costlessly dispose in market $B$ under particular demand and cost conditions. Suppose firm 1 is maximizing its profits by costlessly disposing in market $B$ (so that $q_1^B < q_1^A = q_1^A$). In this case, firm 1’s profit from market $A$ is $(\alpha - q_1^A - c) \cdot q_1^A$ so that solving the first order condition yields the well-known monopoly solution of $q_1^A = (\alpha - c) / 2$ and $p_A = (\alpha + c) / 2$. Given that firm 1 is costlessly disposing in market $B$, it would incur no cost to expand sales in market $B$ on the margin. Thus, profits accruing to firm 1 and firm 2 from market $B$ are $\pi_1^B = (\alpha - q_1^B - q_2^B) \cdot q_1^B$ and $\pi_2^B = (\alpha - q_1^B - q_2^B - c) \cdot q_2^B$. The respective first order conditions yield the optimal output quantities:

$q_1^B = (\alpha + c) / 3$ and $q_2^B = (\alpha - 2c) / 3$. Finally, by definition, firm 1 costlessly disposes in market $B$ only if $q_1^B < q_1^A$, which requires that $(\alpha + c) / 3 < (\alpha - c) / 2$. Solving for the requisite cost condition reveals that firm 1 costlessly disposes in market $B$ only if $c < \alpha / 5$.

**The results under Cournot Competition with costless disposal (Cournot, CD) in market $B$**

are as follows. Firm 1 costlessly disposes in market $B$ when $c < \alpha / 5$. Equilibrium quantities, prices, and profits are:

$q_1^A = \frac{\alpha - c}{2}$; $q_1^B = \frac{\alpha + c}{3}$; $q_2^A = 0$; $q_2^B = \frac{\alpha - 2c}{3}$; $p_A = \frac{\alpha + c}{2}$; $p_B = \frac{\alpha + c}{3}$;

$$\pi_{1, \text{Cournot, CD}} = \frac{(\alpha - c)^2}{4} + \frac{(\alpha + c)^2}{9}; \text{ and } \pi_{2, \text{Cournot, CD}} = \frac{(\alpha - 2c)^2}{9}.$$
6. Choosing to not Costlessly Dispose

Not accounting for possible limit pricing on the part of firm 1, it has been shown that firm 1 behaves as a dual monopolist if $c \geq 2\alpha / 3$, chooses to compete with firm 2 and without costlessly disposing of output in market $B$ when $\alpha / 5 \leq c < 2\alpha / 3$, and chooses to costlessly dispose of some of its output in market $B$ if $c < \alpha / 5$. The equilibrium quantities, prices, and profits have yet to be determined in the range in which the firms compete without costless disposal.

Since firm 1 does not costlessly dispose in market $B$, $q_A^1 = q_B^1 = q_1$. The profit functions for the firms are then $\pi_1 = (p_A^A + p_B^B - c) q_1 = (2\alpha - 2q_1 - q_2 - c) q_1$ and $\pi_2 = (p_B^B - c) q_2 = (\alpha - q_1 - q_2 - c) q_2$.

Using the first order conditions to find best response functions and solving for the optimal quantities yields $q_1 = (3\alpha - c) / 7$ and $q_2 = (2\alpha - 3c) / 7$.

**The results under Cournot Competition without costlessly dispose (Cournot, NoCD), are as follows.** Assuming that firm 1 does not engage in limit pricing, it will choose to not costlessly dispose in market $B$ when $\alpha / 5 \leq c < 2\alpha / 3$. Equilibrium quantities, prices, and profits are:

\[
q_A^1 = q_B^1 = \frac{3\alpha - c}{7}; \quad q_A^2 = 0; \quad q_B^2 = \frac{2\alpha - 3c}{7}; \quad p_A^A = \frac{4\alpha + c}{7}; \quad p_B^B = \frac{2\alpha + 4c}{7};
\]

\[
\pi_{1\text{Cournot, NoCD}} = \frac{2(3\alpha - c)^2}{49}; \quad \text{and} \quad \pi_{2\text{Cournot, NoCD}} = \frac{(2\alpha - 3c)^2}{49}.
\]

7. Limit Pricing

Firm 1 engages in limit pricing by increasing output in market $B$ in order to keep firm 2 from selling any output in market $B$. In the standard Cournot problem (1 market, 2 firms), neither firm benefits from lowering its price (by over producing) in a static model, because doing so invokes a cost on the firm in terms of foregone profit with no countering benefit. In a dynamic model, limit pricing and other predatory behavior can be profitable if it allows the predatory firm to capture future profits. In the problem considered here, firm 1 may have an incentive to engage in limit pricing that does not stem from

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7 These results hold for $c$ in the range $\alpha / 5 \leq c < 9\alpha / 17$. The possibility of limit pricing is taken up in section 7. In particular, firm 1 engages in limit pricing (in order to drive firm 2 out of market $B$) if $9\alpha / 17 \leq c < 2\alpha / 3$. 

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a dynamic extension.\textsuperscript{8} Rather, because firm 1’s joint production technology allows it to distribute production costs over two markets, it may be optimal for firm 1 to sell more in market \( B \) than it otherwise would (and to receive lower profits in market \( B \) than it otherwise would) in order to receive higher profits in market \( A \).\textsuperscript{9}

Firm 1 engages in limit pricing by increasing output in the \( B \) market in order to drive the price in market \( B \) low enough to just deter entry by firm 2, which requires driving \( p_B \) down to \( c \). Thus, limit pricing requires \( q_1^B = \alpha - c \) so that \( p_B = c \) and \( q_2^B = 0 \). It follows immediately that neither firm receives positive profit in the \( B \) market when firm 1 engages in limit pricing. Moreover, notice that limit pricing results in the efficient quantity being sold in market \( B \). Next, knowing firm 1’s quantity in the \( B \) market under limit pricing allows us to determine firm 1’s behavior in market \( A \).

Figure 1 shows firm 1’s possible decisions under limit pricing. First, as said above, firm 1 engages in limit pricing by increasing sales in market \( B \) so that the price in market \( B \) falls to \( c \). This means that for any \( c \) between 0 and \( \alpha \), \( q_1^B \) is found by extending a horizontal line from \( c \) (on the \( y \)-axis) over to the demand function. Figure 1 shows this explicitly for two hypothetical costs, \( c_H \) and \( c_L \), which are associated with production levels of \( q_{c_H}^B \) and \( q_{c_L}^B \).

Given identical demand functions for markets \( A \) and \( B \), Figure 1 can now be used to determine firm 1’s optimal behavior in market \( A \) by recognizing that firm 1’s marginal cost in market \( A \) equals 0 as long as \( q_1^A \leq q_1^B \) and equals \( c \) if \( q_1^A > q_1^B \). Suppose \( c < \alpha / 2 \) so that \( q_1^B > \alpha / 2 \). In this case, firm 1 behaves as if marginal cost in market \( A \) equals 0 and therefore increases sales until firm 1’s marginal revenue from market \( A \) equals zero (i.e., \( MR_1^A = 0 \)), selling \( q_1^A = \alpha / 2 \), and costlessly disposing of the remaining output. Accordingly, firm 1 receives profit of \( x_1^{Limit Pricing,CD} = \alpha^2 / 4 \). Alternatively, when \( c > \alpha / 2 \), firm 1 sets \( q_1^B < \alpha / 2 \). In this case, firm 1’s marginal cost in market \( A \) is equal to 0 for all units up

\textsuperscript{8} There are legal issues surrounding limit pricing and other possibly predatory behaviors that we currently ignore.

\textsuperscript{9} Clearly firm 1 has no incentive to engage in limit pricing if it is already a dual monopolist, i.e., if \( c \geq 2\alpha / 3 \).
to $q_1^B$ and $c$ afterwards. Thus, when $c \geq \alpha/2$, Figure 1 shows that firm 1’s marginal revenue in market $A$ is above 0 but below $c$ at $q_1^B$. Accordingly, firm 1’s optimal behavior when engaged in limit pricing is to set $q_1^d$ equal to $q_1^B$ (which is equal to $\alpha-c$). In this case, firm 1 is not costlessly disposing in either market, and its profits are $\pi_{1,\text{LimitPricing, NoCD}} = c(\alpha-c)$.\(^\text{10}\)

At this point, we must compare profits under limit pricing vs. not limit pricing. There are three ranges to check:

- $0 \leq c < \alpha/5$ when a firm not engaged in limit pricing would choose to costlessly dispose in market $B$ while a firm engaged in limit pricing would choose to costlessly dispose in market $A$.

- $\alpha/5 \leq c < \alpha/2$ when a firm not engaged in limit pricing would choose to not costlessly dispose but a firm engaged in limit pricing would costlessly dispose in market $A$.

- $\alpha/2 \leq c < 2\alpha/3$ when neither a limit-pricing nor a non-limit-pricing firm would choose to costlessly dispose.

When $0 \leq c < \alpha/5$,

$$\frac{\pi_{1,\text{Cournot,CD}} - \pi_{1,\text{LimitPricing,CD}}}{\pi_1} = \frac{(\alpha-c)^2}{4} - \frac{(\alpha+c)^2}{9} - \frac{\alpha^2}{4}.$$  

The first derivative of this difference with respect to $c$ shows that the difference is always decreasing as long as $c < 5\alpha/13$. Moreover, when $c = \alpha/5$, the difference equals $7\alpha^2/100$, which is positive. Therefore, when $0 \leq c \leq \alpha/5$, firm 1 maximizes it profits by competing with firm 2 in market $B$ and not by limit pricing.

When $\alpha/5 \leq c < \alpha/2$

$$\frac{\pi_{1,\text{Cournot, No CD}} - \pi_{1,\text{LimitPricing, CD}}}{\pi_1} = \frac{2(3\alpha-c)^2}{49} - \frac{\alpha^2}{4}.$$  

The first derivative shows that this difference is decreasing as long as $c < 3\alpha$, which is always the case in the relevant range. Moreover, when $c = \alpha/2$, the difference equals $\alpha^2/196$, which is positive. Therefore,\(^\text{11}\)

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\(^{10}\) Notice that while firm 1 might costlessly dispose in market $B$ when it competes with firm 2 in market $B$, if firm 1 limit prices in market $B$ then it would only consider costlessly disposing in market $A$. 

\(^{11}\) Notice that while firm 1 might costlessly dispose in market $B$ when it competes with firm 2 in market $B$, if firm 1 limit prices in market $B$ then it would only consider costlessly disposing in market $A$. 

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when $\alpha / 5 \leq c < \alpha / 2$, firm 1 maximizes its profits by competing with firm 2 in market $B$ and not by limit pricing.

When $\alpha / 2 \leq c < 2\alpha / 3$,

$$\frac{\pi_{1,\text{Cournot, No CD}}}{\pi_1} = \frac{\pi_{1,\text{Limit Pricing, No CD}}}{\pi_1} = \frac{2(3\alpha - c)^2}{49} - c \cdot (\alpha - c).$$

It is easy to verify that this difference is equal to zero when $c = 9\alpha / 17$ and when $c = 2\alpha / 3$. Moreover, the difference is positive when $c < 9\alpha / 17$ and is negative when $9\alpha / 17 < c < 2\alpha / 3$. Therefore, limit pricing is an optimal strategy for firm 1 in this latter range.

The results when firm 1 engages in limit pricing are as follows. Firm 1 chooses to engage in limit pricing when $9\alpha / 17 \leq c < 2\alpha / 3$. Equilibrium quantities, prices, and profits are:

$$q_1^A = q_2^A = \alpha - c; \quad q_1^B = q_2^B = 0; \quad p_1^A = p_2^B = c; \quad \pi_{1,\text{Limit Pricing, No CD}} = c(\alpha - c); \quad \text{and} \quad \pi_{2,\text{Limit Pricing, No CD}} = 0.$$

8. Surplus Considerations

Surplus comparisons for this model are not obvious, because society always benefits from the innovation (i.e., market $A$ provides surplus that wasn’t available before) and the joint production nature of firm 1’s technology makes comparisons across markets difficult. We will restrict our surplus comparisons, therefore, to one special case. The results detailed in Table 1 will be termed the results under the “innovation.” These outcomes will then be compared to a particular hypothetical intervention, such as by the US Department of Justice (DOJ). Specifically, we assume that if the DOJ intervenes in the market, it will do so by splitting the monopoly (firm 1) into two firms – firm $1A$ that has a patent in market $A$ and firm $1B$ that competes with firm 2 in market $B$. Firms $1A$ and $1B$ cannot interact with one another, nor can they take advantage of the joint production technology. Thus, the DOJ protects firm 1’s patent in market $A$, but does not allow firm 1 to benefit from joint production. We will call this the DOJ’s “intervention.”

Before making surplus comparisons, notice that the DOJ’s intervention is fairly crude in that it forces society to bear a cost of $c$ for every unit of output produced in each market. We show below that
tying the DOJ’s hands in this way is bad for consumers and bad for society regardless of $c$. This potential intervention for the DOJ, however, seems to be a good baseline to compare against. If eliminating the joint production benefits accrued to firm 1 makes consumers and society worse off, perhaps the DOJ should not proceed with an antitrust action.

The results under the DOJ’s intervention are the standard duopoly results under Cournot competition. Firm $1A$ produces a quantity of $(\alpha - c) / 2$ and sells each unit at a price of $(\alpha + c) / 2$. Consumer surplus and total surplus in the $A$ market, therefore, are $(\alpha - c)^2 / 8$ and $3(\alpha - c)^2 / 8$ respectively. In the $B$ market, both firms produce a quantity of $(\alpha - c) / 3$ for a total quantity of $2(\alpha - c) / 3$, each unit sells at the market clearing price of $(\alpha + 2c) / 3$. Consumer surplus and total surplus in the $B$ market, therefore, are $2(\alpha - c)^2 / 9$ and $4(\alpha - c)^2 / 9$ respectively.

Holding $\alpha$ fixed at 1, Figures 2 and 3 show the value of consumer surplus and total surplus under the DOJ’s intervention relative to respective surplus values under the innovation. Notice that consumer surplus and total surplus are identical under the DOJ intervention and under the innovation when $c = 0$. This must be the case as $c = 0$ implies that firm 1 does not benefit from joint production directly (which the DOJ prevents) but does benefit from the monopoly (which the DOJ protects). Figures 2 and 3 show that consumers and society are unambiguously better off under the innovation, and increasingly so as $c$ increases. Again, this is because the primary benefit to society from the innovation is that it lowers the total costs of production, which results in larger quantities being produced (and being sold for lower prices).

Notice in Figures 2 and 3 that there is a precipitous drop in relative consumer surplus at $c = 9/17$. This is due to consumers actually benefiting from firm 1 engaging in limit pricing. That is, when $c = 9\alpha / 17$, firm 1 engages in limit pricing by “over-selling” in market $B$ in order to preclude firm 2 from selling in that market. Firm 1 does this as it more than makes up for lower profits in market $B$ by way of larger profits in market $A$. Consumers, however, also benefit (greatly) from this behavior as market $B$ now appears to be competitive with price equaling $c$. Note too that when firm 1 predates (i.e., from $c = 9\alpha / 17$ to $c = 2\alpha / 3$), relative consumer surplus is held fixed at $25 / 72$. 

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9. Changing the Value of the Monopoly Market

One of the more interesting extensions of the model relaxes the assumption that demand is identical in both markets. Although one could allow either market to become bigger than the other (i.e., a greater quantity intercept), such an extension is fairly uninteresting as the monopolist would eventually treat the markets as separate and behave as if all costs are incurred in the larger market when one of the markets gets substantially larger than the other. The more interesting extension is to assume one of the markets becomes more valuable (i.e., a greater price intercept).

Let market $B$ remain unchanged so that inverse demand in market $B$ is $p^B = \alpha - Q^B$. The value of market $A$ is then altered by changing the maximum willingness to pay in market $A$ while holding fixed the maximum amount of output demanded in market $A$ at $\alpha$. This is accomplished by introducing a new parameter, $\gamma$. Inverse demand for market $A$ is then $p^A = \gamma(\alpha - Q^A)$. Notice that $Q^A = \alpha$ when $p^A = 0$. Moreover, the maximum willingness to pay in market $A$ is now $\gamma \alpha$. Thus, market $A$ is the same “size” as market $B$, but it is more valuable than $B$ when $\gamma > 1$ and is less valuable than $B$ when $\gamma < 1$.

9.1. What happens to the threshold between costlessly disposing in market $B$ and not costlessly disposing in market $B$ as market $A$ becomes more valuable? That is, how does the threshold captured by $\alpha / 5$ change as $\gamma$ increases?

When firm 1 costlessly disposes in market $A$, its quantities under the original model are:

$$q_i^A = \frac{\alpha - c}{2} \quad \text{and} \quad q_i^B = \frac{\alpha + c}{3}.$$  

Setting these equal and solving for $c$ yielded the threshold of $\alpha / 5$. Under the new model in which demand for market 1 is extended with the parameter $\gamma$, it is easy to show that the new quantities when firm 1 costlessly disposes in market $A$ are:
\[ q_1^A = \frac{\gamma \alpha - c}{2\gamma} \quad \text{and} \quad q_1^B = \frac{\alpha + c}{3}. \]

Solving for \( q_1^A \geq q_1^B \) yields a threshold necessitating \( c < \frac{\gamma \alpha}{2 + 3\gamma} \). Notice that this threshold equals \( \frac{\alpha}{5} \) when \( \gamma = 1 \). Moreover, this new threshold is always increasing as \( \gamma \) increases – the derivative of the threshold with respect to \( \gamma \) is \( \frac{3\alpha}{(3 + 2\gamma)^2} \), which is always positive for positive values of \( \alpha \).

In sum, as the monopoly market becomes more and more valuable, the range of \( c \) values for which costless disposal in market \( B \) is better for the monopolist than selling equal quantities in the two markets increases.

9.2 \textit{Will firm 1 continue to limit price under any values of } c \textit{if market } A \textit{becomes much more valuable than market } B \textit{(i.e., } \gamma \gg 1)\textit{?}

An easy way to derive the answer is to compare profits under limit pricing to profits under costless disposal. When \( \gamma > 1 \), profits under costless disposal are \( \left[ (\gamma \alpha - c)^2 / 4\gamma \right] + [(\alpha + c)^2 / 9] \) and profits under limit pricing are \( \gamma c (\alpha - c) \). It is easy to show that the difference in profits goes to infinity as \( \gamma \) goes to infinity for all values of \( c \) between zero and \( \alpha \). Therefore, if \( \gamma \) gets large enough, profits under costless disposal in market \( B \) exceed profits from limit pricing in market \( B \). This is not to say that firm 1 will choose to costlessly dispose in market \( B \); rather it demonstrates that firm 1 will not choose to engage in limit pricing.

9.3 \textit{Describe the monopolist’s general behavior as } \gamma \rightarrow 0.\textit{ }

When \( \gamma < 1 \), market \( A \) (the monopoly market for SD plasma) is substantially less valuable than market \( B \) (the Cournot market for red blood cells). The immediate result is that the monopolist is unable to extend its monopoly power in any meaningful way. As shown in section 7, the monopolist engages in limit pricing by “over-selling” in market \( B \) and therefore lowering its profits in that market in order to increase its profits in market \( A \). When market \( A \) is not very valuable (i.e., as \( \gamma \rightarrow 0 \)), however, the benefits from joint production disappear as the profits from market \( A \) go to zero even if costs are
eliminated. Thus, as $\gamma \to 0$, the monopolist will engage in Cournot competition in market $B$ with each firm behaving as if its marginal cost in that market is $c$. Lastly, note that the monopolist will sell an equal amount in market $A$ as it does in market $B$, but this will not contribute significantly to the monopolist’s profits.

The case in which $\gamma$ goes to zero (or at least is substantially less than one) mirrors reality as the market for red blood cells is much larger and more lucrative in the US than is the market for plasma. The results in the preceding paragraph, therefore, are particularly informative in terms of advocating public policy. In particular, because the market for plasma is much smaller and less valuable than the market for red blood cells, the American Red Cross has no incentive to extend its monopoly in the production of plasma in order to prevent competition with Blood Centers of America in the market for red blood cells. Thus, the DOJ has little reason to intervene in either market.

10. Conclusion

We have presented and solved the equilibrium for a model of joint production. Depending on the parameter values, the firm that benefits from joint production may act as a dual monopolist, engage in limit pricing, or compete with a second firm. We have also discussed some implications regarding efficiency, and have set the stage for relaxing the assumption of identical markets. Appendix A contains a sample problem set with answers. Appendix B contains Maple 10 code that duplicates the results contained in the paper.
References


Table 1. Comparison of Results

<table>
<thead>
<tr>
<th>Range of $c$</th>
<th>Cournot Competition, Costless Disposal</th>
<th>Cournot Competition, No Disposal</th>
<th>Limit Pricing</th>
<th>Dual Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1^A$</td>
<td>$(\alpha-c)/2$</td>
<td>$(3\alpha-c)/7$</td>
<td>$(\alpha-c)$</td>
<td>$(2\alpha-c)/4$</td>
</tr>
<tr>
<td>$Q_1^B$</td>
<td>$(\alpha+c)/3$</td>
<td>$(3\alpha-c)/7$</td>
<td>$(\alpha-c)$</td>
<td>$(2\alpha-c)/4$</td>
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<tr>
<td>$Q_2^A$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Q_2^B$</td>
<td>$(\alpha-2c)/3$</td>
<td>$(2\alpha-3c)/7$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Q^A$</td>
<td>$(\alpha-c)/2$</td>
<td>$(3\alpha-c)/7$</td>
<td>$(\alpha-c)$</td>
<td>$(2\alpha-c)/4$</td>
</tr>
<tr>
<td>$Q^B$</td>
<td>$(2\alpha-c)/3$</td>
<td>$(5\alpha-4c)/7$</td>
<td>$(\alpha-c)$</td>
<td>$(2\alpha-c)/4$</td>
</tr>
<tr>
<td>$p^A$</td>
<td>$(\alpha+c)/2$</td>
<td>$(4\alpha+c)/7$</td>
<td>C</td>
<td>$(2\alpha+c)/4$</td>
</tr>
<tr>
<td>$p^B$</td>
<td>$(\alpha+c)/3$</td>
<td>$(2\alpha+4c)/7$</td>
<td>C</td>
<td>$(2\alpha+c)/4$</td>
</tr>
<tr>
<td>$\pi_1$</td>
<td>$(\alpha-c)^2/4 + (\alpha+c)^2/9$</td>
<td>$2(3\alpha-c)^2/49$</td>
<td>$c(\alpha-c)$</td>
<td>$(2\alpha-c)^2/8$</td>
</tr>
<tr>
<td>$\pi_2$</td>
<td>$(\alpha-2c)^2/9$</td>
<td>$(2\alpha-3c)^2/49$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$CS^A$</td>
<td>$(\alpha-c)^2/8$</td>
<td>$(3\alpha-c)^2/98$</td>
<td>$(\alpha-c)^2/2$</td>
<td>$(2\alpha-c)^2/32$</td>
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<tr>
<td>$CS^B$</td>
<td>$(2\alpha-c)^2/18$</td>
<td>$(5\alpha-4c)^2/98$</td>
<td>$(\alpha-c)^2/2$</td>
<td>$(2\alpha-c)^2/32$</td>
</tr>
<tr>
<td>Total CS</td>
<td>$[9(\alpha-c)^2 + 4(2\alpha-c)^2]/72$</td>
<td>$[(3\alpha-c)^2 + (5\alpha-4c)^2]/98$</td>
<td>$(\alpha-c)^2$</td>
<td>$(2\alpha-c)^2/16$</td>
</tr>
</tbody>
</table>
Figure 1. Firm Quantities under Limit Pricing

\[
\alpha = \frac{H}{2} \\
\alpha = \frac{L}{2} \\
q_{c_H} = \alpha - c_H \\
q_{c_L} = \alpha - c_L
\]
Figure 2. Consumer Surplus under a DOJ Intervention relative to under the Innovation ($\alpha = 1$).
Figure 3. Total Surplus under a DOJ Intervention relative to under the Innovation ($\alpha = 1$).
Appendix A. A Problem Set of Joint Production

Denote two firms by subscripts 1 and 2. Denote two markets by superscripts \(A\) and \(B\). Firm 1 is a patent holder in market \(A\), precluding firm 2 from selling any output there. The two firms (potentially) engage in Cournot competition in market \(B\).

The production process is such that one unit of production produces one unit of output to be sold in each market. Thus, firm 1 chooses a production level, \(q_1\), which allows it to sell up to that quantity in both markets. Denote by \(q^A_1\) and \(q^B_1\) the amount of output firm 1 decides to sell in each market subject to the constraints that \(q^A_1 \leq q_1\) and \(q^B_1 \leq q_1\). Total revenue for firm 1 is \(p^A q^A_1 + p^B q^B_1\). Firm 1 faces no fixed costs and a constant marginal cost of production of \(c\) where \(c \geq 0\) such that its total costs are \(cq_1\).

Firm 2’s problem can be explained in an identical fashion. Firm 2 chooses an output level, \(q_2\). Concurrent with making its total production decision, firm 2 also decides how much to sell in the \(B\) market, denoted by \(q^B_2\), subject to the constraint that \(q^B_2 \leq q_2\). As firm 1 is a patent holder in market \(A\), firm 2 doesn’t sell any of its output in market \(A\) (i.e., \(q^A_2 = 0\)). Total revenue for firm 2 is \(p^B q^B_2\). Firm 2 faces no fixed costs and a constant marginal cost of production of \(c\) where \(c \geq 0\) such that firm 2’s total costs are \(cq_2\).

Finally, let total quantity supplied to each market be denoted by \(Q^A\) and \(Q^B\) where \(Q^A = q^A_1 + q^A_2 = q^A_1\) and \(Q^B = q^B_1 + q^B_2\). Assuming that demand in both markets is identical and linear, market clearing prices are \(p^A = \alpha - Q^A\) and \(p^B = \alpha - Q^B\) where \(\alpha \geq c\).

Lastly, it is assumed that either (or both) firms may **costlessly dispose** of units of either good that it elects not to sell. After making its total production decision, for example, firm 1 is said to costlessly dispose in market \(A\) if \(q^A_1 < q^B_1 = q_1\) and is said to costlessly dispose in market \(B\) if \(q^B_1 < q^A_1 = q_1\).

**Part A Questions**

1. Will firm 2 ever find it optimal to costlessly dispose in market \(B\)?

2. Will firm 1 ever find it optimal to costlessly dispose in both markets simultaneously?

3. Find a (the) Nash equilibrium for all values of \(c\) between 0 and \(\alpha\).

**Hints**

There are only two parameters in the model – \(\alpha\) and \(c\). Conceptually, fix \(\alpha\) and let \(c\) range from 0 to \(\alpha\). There are several possible outcomes to consider.

- Conditions may be such that firm 1 can act like a monopolist in both markets because firm 2 wouldn’t find it optimal to sell in market \(B\) even when firm 1 restricts its supply in market \(B\) to the monopoly quantity. When this is the case, we will say that firm 1 is a dual monopolist.

- Conditions may be such that firm 1 sells a positive amount in both markets and firm 2 sells a positive amount in market \(B\). In this situation, it is important to consider how firm 1 would optimally think about its costs (and potentially how to divide the costs across markets). It is also important to determine if firm 1 costlessly disposes in either market.
Finally, firm 1 may find it optimal to engage in limit pricing in market $B$. Limit pricing requires that firm 1 sell so much output in market $B$ that firm 2 chooses to not sell any output in market $B$.

**Part B Questions**

There are two ways in which firm 1 can exploit its monopoly power in this model – under some conditions firm 1 may spread the marginal cost of production across two markets, which gives it an advantage over firm 2 which must recover its marginal cost of production in only one market; under other conditions firm 1 engages in limit pricing, which drives firm 2 from market $B$.

4. Except under special situations, it can be shown that a monopolist cannot extend its monopoly power across markets. This problem, however, is one of those special situations. Explain the economic intuition underlying why firm 1 can exploit its monopoly power by limit pricing in this case.

5a. Suppose the DOJ accuses firm 1 of engaging in predatory behavior (i.e., limit pricing) whenever its quantity in market $B$ is higher than it otherwise would be if firm 1 didn’t benefit from joint production. Under this definition, for what values of $c$ would the DOJ accuse firm 1 of predatory behavior?

5b. Suppose the DOJ accuses firm 1 of engaging in predatory behavior (i.e., limit pricing) whenever price in market $B$ is higher than it otherwise would be if firm 1 didn’t benefit from joint production. Under this definition, for what values of $c$ would the DOJ accuse firm 1 of predatory behavior?

6. Consider a situation in which firm 1 has developed its innovation and holds a patent for the sale of goods in market $A$. Thus, your answer to question 3 holds. The DOJ suspects that firm 1 is extending its monopoly power in a way that hurts consumers (and society). The DOJ’s only possible intervention is to split firm 1 into two firms – firm $1A$ and firm $1B$. Firm $1A$ produces and sells in market $A$, while firm $1B$ produces and sells in market $B$. Firms $1A$ and $1B$ are not allowed to interact, and therefore each faces a marginal cost of production of $c$. In short, the DOJ’s intervention prevents firm 1 from benefiting from its joint production capabilities, but its monopoly right in market $A$ is protected. Provide a graph of consumer surplus summed across the two markets under the DOJ’s intervention relative to the consumer surplus realized when firm 1 is allowed to benefit from joint production. Provide a second graph of total surplus summed across the two markets under the DOJ’s intervention. Discuss what these graphs reveal.

**Part C Questions**

While keeping market $B$ unchanged, so that $p^B = a - Q^B$, let the value of goods in market $A$ vary from those in market $B$ but keep the potential size of both markets unchanged. To do this, define $\gamma > 0$ such that $p^A = \gamma a - \gamma Q^A$. Notice that $Q^A = a$ when $p^A = 0$. Moreover, the maximum willingness to pay in market $A$ is now $\gamma a$. Thus, market $A$ is the same “size” as market $B$, but it is more valuable than market $B$ when $\gamma > 1$ and is less valuable than market $B$ when $\gamma < 1$.

7. What happens to the threshold between costlessly disposing in market $B$ and not costlessly disposing in market $B$ as market $A$ becomes more valuable? That is, how does the threshold captured by $a/5$ change as $\gamma$ increases?

8. Will firm 1 continue to limit price under any values of $c$ if market $A$ becomes much more valuable than market $B$ (i.e., $\gamma \gg 1$)?

9. Describe the monopolist’s general behavior as $\gamma \rightarrow 0$. 

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Answers to the Problem Set

1. Yes, whenever $c < \alpha / 5$. See section 5 of the text.

2. A firm will never find it optimal to costlessly dispose in both markets. Production is costly (marginal cost of $c$), and so if a firm costlessly disposed of $\delta^A$ units in market $A$ and of $\delta^B$ units in market $B$, the firm could increase its profits by producing $\min\{\delta^A, \delta^B\}$ less of overall production.

3. The full solution to the problem is found in sections 3 – 7 of the text.

4. In the right range of $c$, firm 1 will engage in limit pricing in market $B$, precluding firm 2 from selling any output. Notice that firm 1 can always compete ala Cournot in market $B$ with firm 2 and earn positive profits, even if firm 1 acts as if its marginal cost of production in market $B$ is $c$. Thus, when firm 1 limit prices in market $B$, it is losing money by setting the price in market $B$ equal to $c$. Firm 1 might choose to do precisely this, however, if it receives enough additional profit in market $A$. In particular, when the firm limit prices in market $B$, it produces sufficient quantity to meet all demand at price $c$. This allows for the possibility that firm 1’s profits from market $A$ have increased more than its profits have decreased in market $B$.

   This tradeoff centers on the value of the two markets. As $c$ increases, the markets become less valuable as the maximum willingness to pay is fixed at $\alpha$ in both. Increases in $c$, therefore, reduce profits for both firms. This reduction in profits is more strongly felt in market $B$, where the two firms compete. Thus, if $c$ is large enough, the value of market $B$ is quite small. It is in such a case that firm 1 is more likely to find it valuable to forego all profits from market $B$ in order to increase profits in market $A$.

5a. Without the innovation, firm 1 would always compete with firm 2 in market $B$ as long as $c < \alpha$. The standard Cournot solution is that both firms produce $(\alpha - c) / 3$. The “$q_1^B$” row of Table 1 shows that firm 1 always produces more output in market $B$ under the innovation than it would continue to produce if the DOJ split it into two firms. Thus, for all values of $c$, firm 1 would be considered to be engaged in predatory behavior.

5b. Without the innovation, firm 1 would always compete with firm 2 in market $B$ as long as $c < \alpha$. The standard Cournot solution is that both firms would produce $(\alpha - c) / 3$ so that total quantity is $2(\alpha - c) / 3$ and the market clearing price is $(\alpha + 2c) / 3$. The “$p^B$” row of table 1 shows that the price in market $B$ is always lower when firm 1 benefits from the innovation than it would be if the DOJ split firm 1 into two firms. Thus, for all values of $c$, firm 1 would never be considered to be engaged in predatory behavior.

6. See section 8 of the text and figures 2 and 3.

7. See section 9.1 of the text.

8. See section 9.2 of the text.

9. See section 9.3 of the text.