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Inter-firm complementarities in R&D: a re-examination of the relative performance of joint ventures

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Abstract

We compare duopoly outcomes between two alternative modes of research and development (R&D), viz. independent R&D and non-cooperative research joint ventures (RJVs), when there are complementarities between firm-specific R&D resources. When complementarity is high, RJVs lead to higher technological improvement and the reverse holds for low complementarity. In the intermediate range, the comparison depends on the relative imperfection in spillovers afflicting independent R&D. In sharp contrast to results on cooperative RJVs, non-cooperative RJVs lead to higher technological improvement when spillovers affecting independent R&D are *low*; the reverse holds for *high* spillovers. When RJVs yield higher technological improvement, they also yield higher industry profit and social welfare.

JEL classification: D43; L13; L23

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1. Introduction

Research alliances such as research joint ventures (RJVs) among firms who are competitors in the product market are fairly commonplace, particularly in high-tech industries. While these alliances are feared by some as precursors to product

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market collusion, public policy in the US and in Western Europe has consciously moved towards permitting and even actively encouraging them. In most of these countries, governments have been concerned about maintaining and advancing technological superiority of domestic firms in the global market. For policy makers in these countries, one of the compelling arguments was that RJVs achieve greater technological improvement (and in a socially cost-effective manner) relative to situations in which firms carry out their R&D activities independently. This paper re-examines the economic foundations of this argument in the context of complementarity of firm-specific R&D resources or inputs and non-cooperative research joint ventures.

One of the main benefits of RJVs is that they avoid duplication of research activities, thus reducing the social cost of achieving any particular level of R&D output. RJVs also help firms reap scale economies in R&D. However, when firms undertake independent R&D, competitive pressures prompt firms to invest aggressively in their own R&D in order to gain advantage over their rivals so that even if they are not able to reap the cost benefits associated with joint ventures, the level of technological advancement achieved might be more than what would have resulted from a RJV (Brander and Spencer, 1983). The existence of technological spillovers (say, due to imperfect appropriability of patent rights), however, dilutes this strategic incentive of firms to invest aggressively in independent R&D (Spence, 1984), while cooperation in R&D through RJVs can internalize this externality. If technological spillovers are sufficiently large, independent R&D leads to lower technological improvement and welfare relative to RJVs (Katz, 1986; d'Aspremont and Jacquemin, 1988).[1]

There is, however, a basic problem in moving from this last argument to policy support for all RJVs. In the framework adopted in most of these models, when firms set up research joint ventures they invest or commit R&D resources *cooperatively* so as to maximize the joint profits they will derive as future competitors in the product market. However, in reality there is no reason to suppose that such complete binding agreements are in effect or that they are even possible. For one, the contribution of an individual firm to a joint venture is often not just a monetary contribution. Rather, it involves sharing of human capital, accumulated knowledge embedded in firm-specific factors, and access to information and activities within its own R&D division (which, in most cases, deals with a much wider spectrum of projects than the scope of the joint venture). It is difficult to see how firms can write binding contracts on such contributions. It is more realistic to suppose that, at least to a certain extent, participant firms in a

[1] A large class of models has extended these pioneering papers to analyze different aspects of cooperative versus independent R&D. See, among many others, Katz and Ordover (1990), de Bondt and Veugelers (1991), Suzumura (1992), Choi (1993), Gandal and Scotchmer (1993), Vonortas (1994), Ziss (1994), Steurs (1995), and Salant and Shaffer (1998). Uncertainty in R&D production is introduced by Marjit (1991).

RJV decide on their contributions non-cooperatively, i.e. independently and competitively. Following the literature, we call this a '*competitive* RJV'. For a public authority, it would be extremely difficult to verify whether a RJV is, in effect, a cooperative R&D cartel or whether it is a competitive RJV. Hence, policies that allow and promote the formation of RJVs ought to consider the conduct and performance of markets under competitive RJVs.

In an important contribution to the debate, Kamien et al. (1992) compare the technological outcomes from alternative ways of organizing R&D including independent R&D and competitive RJVs — the latter being equivalent to independent R&D with perfect technological spillovers. Using a two-stage simultaneous move game where each firm first decides on its contribution of R&D resource inputs to the joint venture and in the second stage chooses its price or quantity in the product market, the authors show that no matter how large the degree to which technological spillovers afflict independent R&D, '*creating a competitive research joint venture reduces the equilibrium level of technological improvement and increases equilibrium prices compared to when firms conduct R&D independently*'. Thus, unlike cooperative RJV cartels, competitive RJVs do not perform as well as independent R&D. The argument that public policy should promote all RJVs because they secure greater technological progress, therefore, appears to rest on somewhat weak economic foundations; the welfare comparison is also ambiguous.

An important feature of the model analyzed by Kamien et al. (1992) is that, like much of this literature, they ignore an important reason behind the formation of joint ventures — complementarity of firm specific R&D inputs in the R&D process. Firms are not merely technological entities but are rather complex conglomerations of human capital and knowledge accumulated through past learning. The historical path of learning and R&D activities generates *firm-specific human capital, knowledge, and R&D resources*.[2] Even if factors of production are freely traded in the market, not all of these firm-specific resources are available to other firms. Furthermore, past learning and independent research creates divergence in knowledge and expertise of different firms, which are often likely to be complementary. The marginal productivity in terms of R&D output resulting from a firm's investment in an R&D process depends on the extent to which this process can access the specific R&D resources of other firms. This access is perfect in a RJV, while in the case of independent research it depends on the extent of 'spillovers' (which are typically imperfect).

There is extensive evidence of complementarity of resources being one of the most important factors motivating the formation of joint ventures. Analyzing over 7000 co-operative agreements worldwide, Hagedoorn and Schakenraad (1990) report that complementarity is one of the primary motives for the formation of joint ventures and research corporations in information technologies, biotechnol-

[2] See Mookherjee and Ray (1992) for studies on indirect labor learning and organizational capital.

ogy, and 'new materials'; as a motivation for joint R&D projects (indistinguishable from RJVs in our theoretical framework) complementarity is by far the most important motivating factor. For example, for many of the RJVs formed in Japan's new technology industries such as semiconductors, computers, biotechnology and aerospace, a primary motivation was the shortage of qualified R&D personnel within each firm which meant that, irrespective of the level of monetary investment in R&D projects, it was important for the R&D division of each firm to gain access to the human capital of other firms.[3]

In this paper, we generalize the model analyzed by Kamien et al. (1992) by allowing for complementarity between firm-specific R&D resources and compare the performance of *competitive* RJVs relative to independent R&D. The existing literature comparing alternative R&D regimes in oligopoly does contain some attempts to incorporate complementarity. Choi (1992) models complementarity between a contractible and a non-contractible research investment though there is no explicit complementarity of resources across firms. Bensaid et al. (1994) model inter-firm complementarity in a limited way — marginal productivity of investment by a firm depends on the investment made by other firms to the extent that all firms must make a strictly positive investment in order for spillovers to occur; an increase in investment by other firms beyond zero, however, does not directly affect the marginal productivity of investment in R&D by a firm. A more satisfactory treatment is that by Veugelers and Kesteloot (1994, 1996) who analyze stability and endogenous formation of joint ventures. They introduce complementarity through a 'synergy effect' — a fixed mark-up on the marginal productivity of investment in R&D — which arises when R&D is carried out under a RJV *and* member firms do not cheat on the original agreement. Unlike our approach, *marginal* productivity of a firm's input into R&D does not depend on the effective R&D inputs of other firms if there is no RJV or if members do not cooperate.[4] None of the above mentioned papers deal with the question addressed here, viz. the comparison of a competitive RJV and independent R&D.

The two modes of R&D compared in this paper differ in only one aspect, viz. the degree of R&D spillovers. Spillovers are perfect in a RJV while they are generally imperfect in the case of independent R&D.[5] There are two important forces which determine the comparison of R&D output between the two modes. First, under independent R&D, imperfection in spillovers implies higher appropriability of R&D investment relative to a RJV so that the competitive urge to be ahead of one's rival tends to make the incentive to invest in R&D high relative to

[3] See, for example, Sinha and Cusumano (1991) and references cited therein.

[4] Leahy and Neary (1997) also discuss 'synergies' under cooperative R&D ventures but what they really refer to is higher technological spillovers in joint ventures.

[5] Although spillovers, in actuality, may not be perfect in a RJV, it is likely that they are much greater in a RJV than under independent research. Our comparisons between RJVs and independent research rely only on the level of spillovers being greater in a RJV than under independent research.

that in a RJV, where the incentive is further dampened by an extreme free-rider problem. This *appropriability effect* on the relative incentive to invest under independent R&D (vis-a-vis RJVs) is stronger when the spillovers affecting independent R&D are low. Second, in a RJV, spillovers are perfect which means that firms can access all of their rivals' R&D inputs. This tends to make their incentive to invest in R&D, as well as the productivity of R&D investment, high relative to that under independent R&D where spillovers are imperfect (so that a firm's access to its rival's R&D input is only partial). This *complementarity effect* becomes more pronounced as the degree of complementarity between R&D inputs of firms increases.

When the degree of complementarity between R&D inputs of firms is very low, then (as in Kamien et al., 1992 where there is no complementarity) the appropriability effect dominates the comparison of market outcomes between the two modes of R&D so that no matter how high the spillovers, independent R&D leads to greater R&D output, i.e. RJVs achieve a lower level of technological improvement despite the productivity gains from perfect access to rivals' inputs. On the other hand, when the degree of complementarity is very high, the complementarity effect dominates so that no matter how low spillovers are, independent R&D does not lead to as much technological improvement as a RJV even though the latter suffers from a greater free-rider problem.

When the degree of complementarity is moderate but not too low, the comparison of R&D outcomes between the modes of organizing R&D depends on the extent of spillovers. Furthermore, the way the degree of spillovers influences the comparison is exactly the opposite of that obtained in the existing literature on the comparison of *cooperative* RJVs to independent R&D. *For high spillovers, competitive RJVs achieve lower technological improvement compared to independent R&D and exactly the reverse occurs for low spillovers*. The reason why the effect of spillovers on the comparison is the opposite of that obtained in the case of *cooperative* RJVs is rooted in the fact that competitive RJVs cannot internalize spillovers i.e., in the absence of complementarity a competitive RJV does not overcome the appropriability effect (as demonstrated by Kamien et al., 1992). When the degree of complementarity is moderately high, however, a competitive RJV can overcome the appropriability effect when spillovers in independent research are *low* — independent R&D suffers from firms not being able to take much advantage of the productivity gains accruing from their rivals' investments in R&D. When spillovers affecting independent R&D are high, both independent research and competitive RJVs are able to take advantage of the complementarity and, as there is greater appropriability of the returns to investment under independent R&D, the latter creates greater technological output.

Finally, we show that when competitive RJVs attain higher technological improvement relative to independent R&D, they also lead to higher profits and social welfare. That is, the social and private incentives to forming a RJV are both positive here. In addition, there exists a region of the parameter space where independent R&D leads to more technological improvement but RJVs attain higher social welfare and profit. We illustrate the profit and welfare comparison with an example.

Section 2 contains the model and some preliminaries. Section 3 discusses the main results comparing the technological performance between competitive RJVs and independent R&D. Section 4 contains the comparison of social welfare and industry profit. Section 5 concludes. All proofs are contained in Appendix B.

2. Model

Consider a symmetric linear Cournot duopoly where the inverse market demand function $P(Q)$ is given by:

$$P(Q) = a - Q, \quad 0 \leqslant Q \leqslant a. \tag{1}$$

There are two identical firms in the market each producing output at constant unit cost. Initially, each firm's unit cost of production is c, where $0 < c < a$. Prior to product market competition, each firm has an option of carrying out R&D in order to reduce its unit cost of production. The output of the R&D process (knowledge created) available to firm i will be denoted by k_i which in turn reduces the unit cost of production for firm i to:

$$c_i = c - f(k_i) \tag{2}$$

where f is an increasing, differentiable and concave function satisfying certain restrictions that we specify later in this section. We consider two alternative regimes under which R&D is carried out in this industry:

(i) *Independent R&D*: where the firms carry out their R&D activities separately, though the R&D process of one firm may generate technological spillovers which enter as a complementary input into the R&D production process in the other firm. The R&D outputs (k_1 and k_2) may differ across firms.

(ii) *RJVs*: where the two firms form a competitive research joint venture and gain identical R&D output, k.

Whether R&D is carried out in a single firm or in a joint venture, the output of any R&D process depends on the *effective inputs of R&D resources* of *both* firms. For $i = 1, 2$, let $x_i \geqslant 0$ denote actual input of R&D resources by firm i. Without loss of generality we set the unit cost of R&D resource input to equal 1. When firms carry out R&D in a RJV, the joint venture has perfect access to the research resource input of each firm. When firms carry out R&D independently, however, the R&D division of firm i has perfect access to its own R&D resource input but not to that of the other firm. In this case, for the R&D production process in firm i, the effective research input from rival firm j, $j \neq i$, is given by βx_j where $\beta \in [0, 1]$ is the technological spillover parameter. If $\beta = 0$, there are no spillovers so that the R&D production process within a firm does not have any access to the R&D

resources of its rival firm. On the other hand, if $\beta = 1$, we have the case of perfect spillovers so that each firm's R&D division has perfect access to the R&D resources invested by its rival firm and the outcome is, in fact, identical to that achieved in the case of a RJV.

Thus far, the setup of our model is identical to that in Kamien et al. (1992). The new element introduced in this paper is the complementarity between the effective R&D resource inputs of the two firms in the R&D production technology. We model a typical R&D production process (carried out in a joint venture or independently) in the following way. Let z_i, $i = 1, 2$, denote the *effective R&D resource input* from firm i entering this production process. In a RJV, $z_i = x_i$, $i = 1, 2$. In the case of independent R&D, for the purpose of R&D production in firm i, $z_i = x_i$, but $z_j = \beta x_j$, $j \neq i$. The R&D output, generated by this process is given by the following CES production function:

$$[(z_1)^\rho + (z_2)^\rho]^{1/\rho}, \quad 0 \leqslant \rho \leqslant 1. \tag{3}$$

The parameter ρ indicates the degree of complementarity between the R&D resources of the two firms. A lower value of ρ implies a higher degree of complementarity. The existing literature, including the paper by Kamien et al. (1992), assumes perfect substitutability by setting ρ equal to 1 so that the marginal productivity of R&D investment of each firm is always independent of the investment made by the other firm. Observe that complementarity implies that the R&D production technology also exhibits superadditivity, viz. $[(z_1)^\rho + (z_2)^\rho]^{1/\rho} > z_1 + z_2$, except for the case where $\rho = 1$ in which case they are equal.

Like much of the existing literature, we model competition between the firms as a two stage game. In the first stage, firms simultaneously decide on their R&D resource inputs x_1 and x_2 which, depending on the way R&D is organized, generates R&D output levels k_1 and k_2. In the second stage, firms simultaneously choose their output, q_1 and q_2. The profit of firm i is then given by:

$$\pi_i = [a - q_1 - q_2 - (c - f(k_i))]\, q_i - x_i. \tag{4}$$

We focus on the (symmetric) subgame perfect equilibria of this game.

Consider the last stage of the game with a given set of R&D output levels, k_1 and k_2. The following conditions on f ensure that there is always a well-defined and unique Cournot–Nash equilibrium in the product market (see Appendix A for details):

$$f(0) = 0, \quad 0 \leqslant f(k) \leqslant c \ \text{ for all } \ k \geqslant 0 \tag{5}$$

and, further,

$$\lim_{k \to \infty} f(k) < a - c. \tag{6}$$

Note that as f is concave and increasing, (5) implies that

$$\lim_{k\to\infty} f'(k) = 0. \tag{7}$$

The (reduced form) profit for firm i, $i=1, 2$, from the continuation game in the second stage is then:

$$\pi_i = \left(\frac{a - c + 2f(k_i) - f(k_j)}{3}\right)^2 - x_i \tag{8}$$

where it is understood that k_1 and k_2 are determined by x_1 and x_2 according to whichever of the two R&D production modes (described earlier) is in place. Furthermore, the following condition ensures that the technology of cost reduction through R&D is initially productive enough to prompt both firms to invest a strictly positive amount of R&D resources (see Appendix A for details):

$$f'(0) > \frac{9}{2(a-c)}. \tag{9}$$

In combination with Eqs. (5)–(7), Eq. (9) ensures that both in the case of a RJV and in the case of independent R&D, there is an interior equilibrium in the (reduced form) R&D investment game where both firms invest equal and positive amounts of R&D resources.

Finally, we assume that:

Assumption (*). $G(k) = [a - c + f(k)]f'(k)$ is decreasing in k.

Assumption (*) requires that, when both firms make symmetric investments in R&D, the marginal profit in the continuation game from any addition to the knowledge created by R&D decreases as the level of such knowledge increases. It is satisfied when f is 'sufficiently concave'. This assumption ensures the existence of a *unique* symmetric equilibrium in the reduced form R&D game (for both a RJV and independent R&D) and plays an important role in the comparison of equilibrium outcomes under different modes of R&D. Note that all of the restrictions imposed on f (including assumption (*)) are identical to the assumptions made in Kamien et al. (1992).[6]

3. Comparison of R&D outcomes

In this section we discuss the equilibrium outcomes of the reduced form first stage game where firms decide on their input of R&D resources and the payoffs

[6] Kamien et al. (1992) allow for more than two firms and for product differentiation. When their model is restricted to two firms ($n=2$) and no product differentiation ($\gamma=1$) their assumptions coincide with ours except for our CES R&D production function, Eq. (3), which is a generalization of the one in their paper in order to allow for complementarity.

are given by the Cournot–Nash profits described by (8). The main question addressed here concerns the comparison of the level of technological improvement (R&D output) generated in the two alternative modes of R&D, i.e. a competitive RJV versus independent R&D.

3.1. RJV competition

First, consider the case where firms carry out their R&D in a competitive RJV so that the R&D output generated is identical for both firms. In particular, if firm i's input of R&D resources to the joint venture is x_i, $i=1, 2$, then each firm gains R&D output equal to:

$$k_1 = k_2 = k(x_1, x_2) = [(x_1)^\rho + (x_2)^\rho]^{1/\rho}. \tag{10}$$

Using (8), the Cournot–Nash profit of firm i is given by:

$$\pi_i(x_1, x_2) = \left(\frac{a - c + f(k(x_1, x_2))}{3}\right)^2 - x_i. \tag{11}$$

For either firm, the marginal return from investing R&D resources in the joint venture is given by:

$$\frac{\partial \pi_i(x_1, x_2)}{\partial x_i} = \frac{2}{9}[a - c + f(k(x_1,x_2))]\frac{\partial f(k)}{\partial k}\frac{\partial k(x_1, x_2)}{\partial x_i} - 1. \tag{12}$$

Appendix A shows that, under the assumptions made on f, there exists a unique Nash equilibrium in the reduced form R&D input game and in this equilibrium, firms set $\hat{x}_1 = \hat{x}_2 = \hat{x} > 0$ such that:

$$\frac{\partial \pi_i}{\partial x_i}\Big|(x_i = \hat{x}, x_j = \hat{x}) = 0 \tag{13}$$

and the equilibrium R&D output of the joint venture is given by:

$$\hat{k}_1 = \hat{k}_2 = \hat{k} = \hat{m}\hat{x} \tag{14}$$

where $\hat{m} = 2^{1/\rho}$. Using (10), (12) and (14), the equilibrium condition in (13) can be written as:

$$G(\hat{k}) = 9/(2\hat{m}^{1-\rho}) \tag{15}$$

where, as defined in the previous section, $G(\hat{k}) = [a - c + f(\hat{k})]f'(\hat{k})$.

3.2. Competition under independent R&D

Next, we consider the equilibrium of the reduced form R&D game when firms carry out their research independently. In this case, if firm i's input of R&D resource is x_i, $i=1, 2$, then the realized R&D output for firm i is given by:

$$k_i = k_i(x_i, x_j) = [(x_i)^\rho + (\beta x_j)^\rho]^{1/\rho}, \quad i, j = 1, 2, \quad i \neq j. \tag{16}$$

Using (8), the Cournot–Nash profit of firm i is given by:

$$\pi_i(x_i, x_j) = \left(\frac{a - c + 2f(k_i(x_i, x_j)) - f(k_j(x_j, x_i))}{3} \right)^2 - x_i, \quad i, j = 1, 2, \quad i \neq j \tag{17}$$

Again, given the symmetric structure of the model and under the assumptions made on f, Appendix A contains the details showing that there exists a unique, symmetric Nash equilibrium to the reduced form game where firms invest equal amount of R&D resources, $\tilde{x}_1 = \tilde{x}_2 = \tilde{x} > 0$ such that:

$$\frac{\partial \pi_i}{\partial x_i} \bigg| (x_i = \tilde{x}, x_j = \tilde{x}) = 0 \tag{18}$$

and the resulting R&D output gained by each firm is given by:

$$\tilde{k}_1 = \tilde{k}_2 = \tilde{k} = \tilde{m}\tilde{x} \tag{19}$$

where $\tilde{m} = (1 + \beta^\rho)^{1/\rho}$. The equilibrium condition in (18) can be written as:

$$G(\tilde{k}) = \frac{9}{2(2 - \beta^\rho)\, \tilde{m}^{1-\rho}} \tag{20}$$

where, as before, $G(\tilde{k}) = [a - c + f(\tilde{k})]\, f'(\tilde{k})$.

3.3. Comparison of R&D output

We are now in a position to address the central issue, i.e. how does the level of technological improvement achieved under a competitive RJV compare to that under independent R&D? In other words, how does $\hat{k}$ compare to $\tilde{k}$?

Under assumption (*), G is a decreasing function. Hence a comparison of (15) and (20) directly yields the following characterization:

Proposition 1. *$\hat{k} \geqslant \tilde{k}$ if and only if $C \leqslant 1$ where $C = (2 - \beta^\rho)(\tilde{m}/\hat{m})^{1-\rho}$.*

The term C summarizes the interaction between the technological spillovers, β, and the degree of complementarity between the R&D resources of the two firms, ρ. Observe that C is *independent* of the choice of f, i.e. the way in which R&D output reduces unit cost does not directly enter into the comparison of research output resulting from the two alternative modes of R&D. As the two modes of

R&D are identical when spillovers are perfect, in the rest of this section and the next section we assume that $\beta < 1$.

The two modes of R&D — competitive RJVs and independent R&D — differ in only one aspect, viz. the degree of spillovers. As pointed out by Kamien et al. (1992), under independent R&D, imperfection in spillovers implies higher appropriability of R&D investment relative to competitive RJVs so that the competitive effect of wishing to be ahead of one's rival tends to make the incentive to invest in R&D higher under independent R&D relative to that in a RJV. The strength of this effect depends on how low the spillover parameter β is and we refer to this as the *appropriability effect*. However, there is another important effect which influences the comparison between the two modes of R&D. In a RJV, spillovers are perfect which means that firms can access all of their rivals' R&D inputs which has a positive effect on their incentive to invest in R&D as well as the productivity of R&D investment (relative to that under independent R&D where spillovers are imperfect). This effect arises purely due to complementarity between R&D inputs of firms — we refer to this as the *complementarity effect*. The lower ρ is, the more pronounced the complementarity effect is likely to be.

In the existing literature, ρ is assumed to equal 1 in which case $C = (2 - \beta) > 1$, so that $\tilde{k} > \hat{k}$, i.e. independent of the spillover parameter β, a competitive RJV leads to inferior outcomes in terms of the level of technological improvement achieved compared to independent R&D; this is the result reported in Kamien et al. (1992). We show that this continues to hold as long as ρ is greater than $\frac{1}{2}$, i.e. the degree of complementarity is sufficiently low. For low complementarity, the appropriability effect dominates the complementarity effect in the comparison of market outcomes between the two modes of R&D so that no matter how high the spillovers, independent R&D leads to greater R&D output.

Proposition 2. *For* $\rho \in (\frac{1}{2}, 1]$, $C > 1$ (*i.e.* $\tilde{k} > \hat{k}$) *for all values of* $\beta \in [0, 1)$, *i.e. a competitive RJV leads to a lower level of R&D output and cost reduction compared to independent R&D (as long as spillovers are imperfect).*

On the other hand, when the degree of complementarity is high enough, the complementarity effect dominates the appropriability effect so that no matter how low spillovers are, independent R&D does not lead to as much technological improvement as a RJV even though the latter suffers from a greater free-rider problem. The fact that firms have greater access to R&D resources of other firms in a RJV outweighs the relatively higher free-rider problem affecting it. Our next proposition shows that when ρ is less than $\frac{1}{3}$, we have a complete reversal of the outcome mentioned in Proposition 2.

Proposition 3. *For* $\rho \in (0, \frac{1}{3}]$, $C < 1$ (*i.e.* $\hat{k} > \tilde{k}$) *for all values of* $\beta \in [0, 1)$, *i.e. a* *competitive RJV leads to a higher level of R&D output and cost reduction compared to independent R&D (as long as spillovers are imperfect).*

Finally, we consider values of ρ lying between $\frac{1}{3}$ and $\frac{1}{2}$, i.e. when the degree of complementarity is neither too high nor too low. In this region, spillovers emerge as important elements in determining the R&D outcome.

Proposition 4. *For each* $\rho \in (\frac{1}{3}, \frac{1}{2})$, *there exists a critical spillover level* $\beta(\rho) \in (0, 1)$ *such that* $C < 1$ *for all values of* $\beta \in [0, \beta(\rho))$ *and* $C > 1$ *for all for all values of* $\beta \in (\beta(\rho), 1)$, *i.e. a competitive RJV leads to a higher level of R&D output and cost reduction compared to independent R&D when the degree of spillovers under independent R&D is lower than* $\beta(\rho)$ *and the reverse happens when the degree of spillovers is higher than* $\beta(\rho)$. *The critical spillover level* $\beta(\rho)$ *falls from* 1 *to* 0 *as* ρ *increases from* $\frac{1}{3}$ *to* $\frac{1}{2}$, *i.e. as the degree of complementarity increases, the technological outcome in a RJV dominates that under independent R&D for higher values of the spillover parameter.*

Since the early papers by Katz (1986) and d'Aspremont and Jacquemin (1988) comparing R&D output under *co-operative* RJVs and independent R&D, there has been a fair degree of consensus that RJVs lead to higher R&D output and welfare if the spillovers under independent R&D are high because cooperative RJVs internalize such spillovers; the reverse may hold when spillovers are low enough as the strategic incentive to invest to be ahead of competitors may dominate the spillover generated disincentive. Kamien et al. (1992) showed that when the RJV is a non-cooperative venture, it always leads to lower R&D output no matter the extent of spillovers. When we introduce complementarity between R&D resources of firms, we see that this is not necessarily true. Furthermore, unless the degree of complementarity is too high or too low, the extent of spillovers is an important factor in the comparison of technological outcomes under a non-cooperative RJV and independent R&D. However, *though technological spillovers matter, their effect is exactly the reverse of what has been obtained in the literature comparing cooperative RJVs to independent R&D: competitive RJVs tend to dominate independent R&D in terms of technological improvement when spillovers are relatively low while the reverse is true when spillovers are high*. The explanation is straightforward. For $\rho \in (\frac{1}{3}, \frac{1}{2})$, the degree of complementarity is quite substantial and being able to take advantage of it, by accessing one's rival's R&D investment, is of great importance. When spillovers are high this is possible for both modes of R&D, and in this case independent research results in more R&D output because of greater appropriability of the returns to investment in R&D. When spillovers are low, however, despite a strong appropriability effect, firms engaged in independent research cannot access much of their rival's R&D investment and hence cannot reap the benefits of complementarity. In this case, therefore, a RJV leads to greater technological output compared to independent research.

Proposition 4 also shows that as the degree of complementarity increases, the range of the spillover parameter over which RJVs lead to higher technological improvement expands — as ρ decreases from $\frac{1}{2}$ to $\frac{1}{3}$, the range increases from zero to the entire unit interval. Fig. 1 summarizes our results on the comparison of R&D output in the (ρ, β) parameter space. Obviously, both modes of R&D yield exactly identical output along the horizontal line at $\beta = 1$. If we confine attention to values of $\beta < 1$, then the $\beta(\rho)$ line in the middle of the box represents all (ρ, β) pairs for which a non-cooperative RJV and independent research produce identical amounts of technological improvement, i.e. all (ρ, β) pairs for which $\hat{k} = \tilde{k}$ (and $C = 1$). On this line, therefore, the cost of R&D inputs is less (i.e. $\hat{x} < \tilde{x}$) in a RJV than under independent research as the RJV takes advantage of the cost savings from having access to complete spillovers. Region A corresponds to higher levels of technological improvements under a RJV than under independent research, while region B is associated with higher R&D output being produced under independent research.

While our model is a direct extension of that in Kamien et al. (1992), our

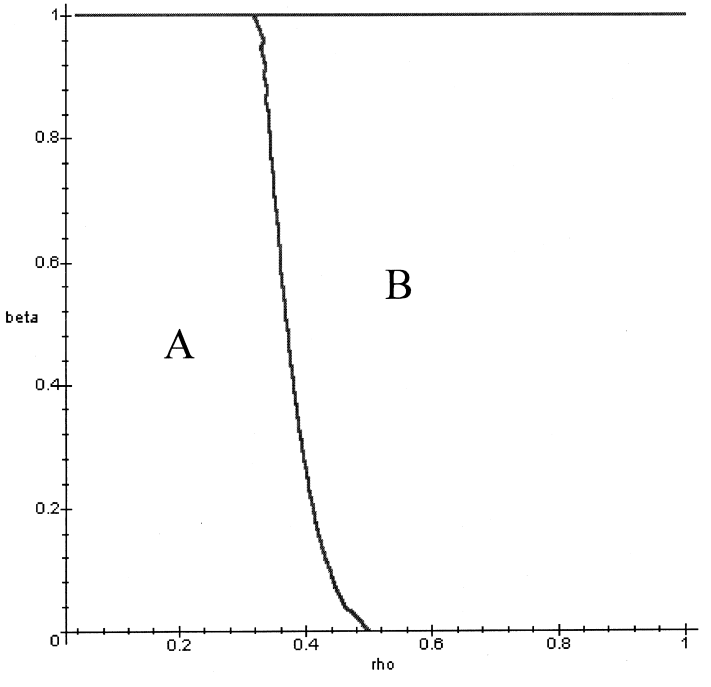

Fig. 1. $C = 1$. Discussion: In region A, a RJV produces more R&D output than independent research (i.e. $\hat{k} > \tilde{k}$). In region B, more technology is produced under independent research than under a RJV.

qualitative results are robust to an alternative way of modeling spillovers and R&D processes, viz. that contained in d'Aspremont and Jacquemin (1988).[7]

4. Private and social incentives

In the previous section, we compared the performance of the two modes of R&D in terms of technological improvements. This enquiry was motivated primarily by the fact that public policy promoting RJVs in many countries is based on the perception that RJVs promote faster technological progress among domestic firms. It is easy to see that, in our context, higher technological progress translates into lower prices, higher output and higher consumer surplus. However, the mode of R&D encouraging higher R&D output may also induce lower duopoly profits and, in fact, lower social welfare.[8] In this section, we try to shed some light on the comparison of industry profit and social welfare between the two modes of organizing R&D. As firms are symmetric, lower industry profit for RJVs would lead to less of a private incentive to form a RJV, while lower social welfare from RJVs would imply that public policy has no reason to promote RJVs.

Consider parameter values where a RJV and independent R&D yield identical levels of technological improvement, i.e. $\hat{k}=\tilde{k}$ (Fig. 1). For these points, the realized unit costs of the two firms are identical across both modes of R&D; the only difference being that firms have incurred lower expenditure on R&D resources under a RJV compared to independent R&D as $\hat{x}<\tilde{x}$. Thus, profits and social welfare are higher under a RJV than under independent research in such situations. Our next proposition shows that this cost saving aspect of doing R&D in a joint venture applies to the entire region of the parameter space where RJVs yield higher R&D output (region A in Fig. 1) so that in this region profits and social welfare are also greater under RJVs than under independent research.

Proposition 5. *Whenever a competitive RJV leads to greater technological improvement than independent R&D, it also yields higher firm profits and generates greater social welfare.*

Thus, to the extent that public policies promote the formation of RJVs on

[7] Amir (2000) demonstrates that the way of modeling the R&D process followed by Kamien et al. (1992) differs fundamentally in some ways from that by d'Aspremont and Jacquemin (1988). One way to adapt our model to the d'Apremont–Jacquemin framework is to interpret x_i as the amount of physical R&D input from firm i entering its R&D production process, leave the CES R&D production function unchanged and incorporate quadratic cost of input x_i. It can be verified that in this case, taking $C=(2-\beta^{\rho})(\tilde{m}/\hat{m})^{2-\rho}$ (which runs from $(\rho=2/3,\ \beta=1)$ to $(\rho=1,\ \beta=0)$), qualitatively similar results to Propositions 1–4 hold.

[8] It is straightforward to show that, for all f satisfying the assumptions of the model, there exists (ρ, β) values for which profits are greater under a RJV than under independent research while R&D output is greater under independent research than under a RJV.

grounds of technological improvement, they would also be making the right decision from the standpoint of social welfare. Moreover, in such cases, private firms will maximize joint profits by being able to form a RJV, i.e. private incentives to form RJVs will be in line with social objectives. Of course, whether firms will actually be able to coordinate their actions and form a RJV depends on other strategic elements which are beyond the scope of the present analysis.

What about the region of the (ρ, β) parameter space, however, where independent R&D yields higher R&D output (region B in Fig. 1)? We have noted above that, when $\hat{k}=\tilde{k}$, RJVs yield higher profits and social welfare and so, by continuity, there is a subset of region B where RJVs continue to yield higher welfare despite independent R&D producing a higher level of R&D output. However, it is likely that in the extreme north-east of the (ρ, β) space — when complementarity is low and spillovers are fairly high — independent R&D will lead to higher profits and welfare. To see why this is likely, we must take into account again the two effects. When complementarity is low (say, for ρ close to 1), the productivity gains from being able to perfectly access rivals' inputs are negligible. Thus, the incentive to invest created by the appropriability problem of spillovers becomes the dominant factor. Furthermore, when spillovers affecting independent R&D are low so that firms are able to appropriate most of the returns on their investment in R&D, competition and the strategic desire to be ahead of rival firms makes each firm invest very aggressively in R&D and in equilibrium, as is well known in the literature, firms overinvest in R&D. This over-investment aspect implies that lower spillovers, while increasing the level of technological improvement, can eventually reduce profit and social welfare. On the other hand, in a RJV, spillovers are perfect and there is a high disincentive to invest due to the appropriability effect — which allows firms to earn greater profits and generate greater social welfare by investing significantly less (creating lower R&D output) than what they would under independent R&D with very low spillovers. When spillovers affecting independent R&D are higher, the strategic incentive to overinvest is dampened and so the profit and welfare performance improves and eventually dominates that under RJVs. We illustrate these features with an example.

Consider $f(k)=c(1-e^{-\lambda k})$ with $\lambda>9/[2c(a-c)]$ (to satisfy Eq. (9)) and $a>2c$ (to satisfy assumption (*)). These parameter restrictions guarantee a unique interior equilibrium in each mode of R&D. Specifically, using Eqs. (15) and (20) one can easily show that, in equilibrium:

$$\hat{k}=-\frac{1}{\lambda}\ln\left\{\frac{a-\sqrt{a^2-\dfrac{18}{\hat{m}^{1-\rho}\lambda}}}{2c}\right\} \quad \text{and}$$

$$\tilde{k}=-\frac{1}{\lambda}\ln\left\{\frac{a-\sqrt{a^2-\dfrac{18}{(2-\beta^{\rho})\,\tilde{m}^{1-\rho}\lambda}}}{2c}\right\}.$$

Fig. 2 plots the $\hat{k}=\tilde{k}$ contour line (left-most), and then, for $a=3$ and $c=1$, the $\hat{W}=\tilde{W}$ (middle) where W denotes social welfare and $\hat{\pi}=\tilde{\pi}$ (right-most) lines are plotted. As shown in the earlier propositions, region A of Fig. 2 yields higher R&D output, profits and social welfare under a RJV compared to independent research. Notice that region A corresponds to either high complementarity or moderate complementarity with relatively low spillovers. In region B, profits and social welfare are greater under a RJV than independent research, even though independent research leads to higher R&D output. Region C, on the other hand, leads to greater R&D output and social welfare under independent research. In region D, where complementarity is low but spillovers are high, R&D output, profits, and welfare are all greater under independent research.

Fig. 3 plots the $\hat{W}=\tilde{W}$ contour for alternative values of λ to show that the

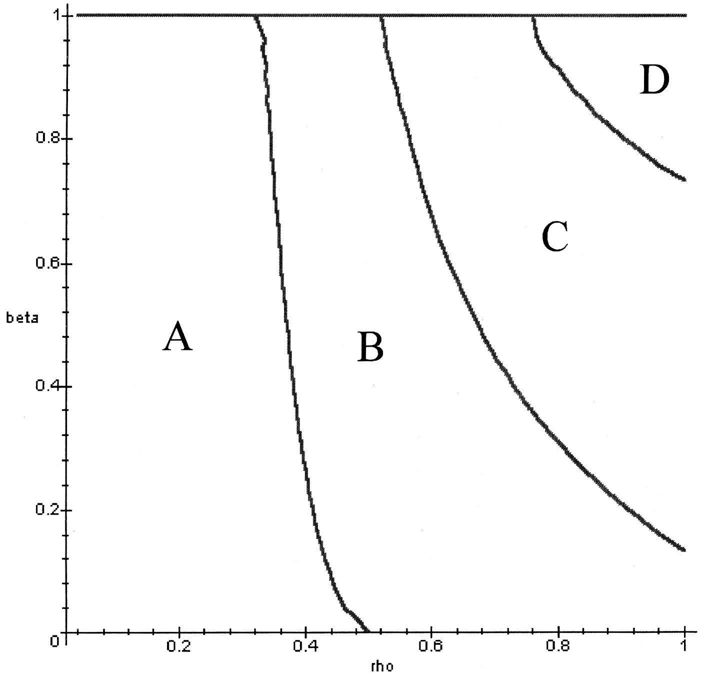

Fig. 2. $C=1$, $\hat{W}=\tilde{W}$, and $\hat{\pi}=\tilde{\pi}$ for $f(k)=c(1-e^{-\lambda k})$. Discussion: Fig. 2 considers an example with $f(k)=c(1-e^{-\lambda k})$, $a=3$, $c=1$, and $\lambda=5$. The line furthest to the left denotes parameter values for which R&D output is identical under a RJV and independent research (i.e. $\hat{k}=\tilde{k}$), as was shown in Fig. 1. R&D output is higher under a RJV in region A and higher under independent research in regions B, C and D. The second line denotes the parameter values for which social welfare is the same under a RJV and independent research (i.e. $\hat{W}=\hat{W}$). Social welfare is greater under a RJV in regions A and B and greater under independent research in regions C and D. The right-most line denotes the parameter values for which profits are the same under a RJV and independent research (i.e. $\hat{\pi}=\tilde{\pi}$). Profits are greater under a RJV in regions A, B and C and greater under independent research in region D.

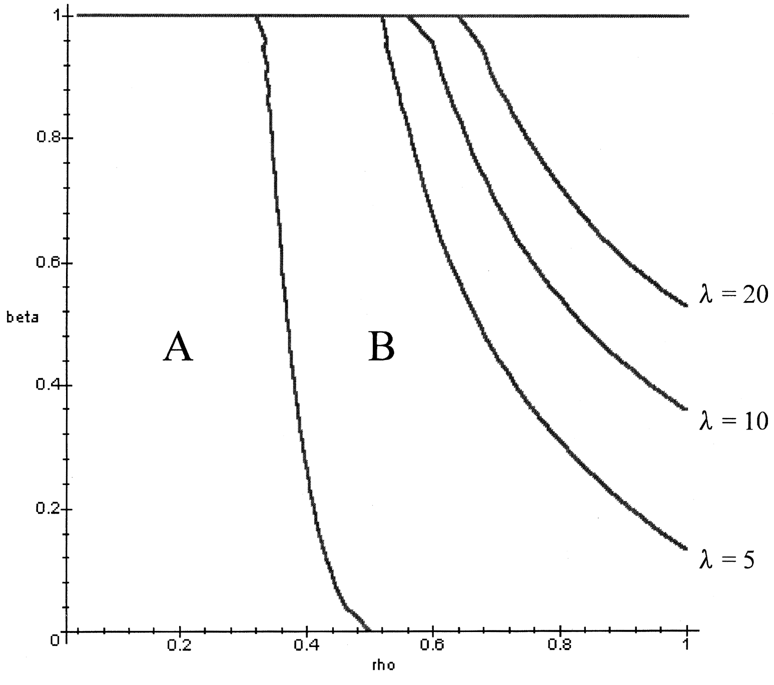

Fig. 3. The effect of cost-effectiveness of R&D on total welfare (exponential example). Discussion: The left-most line denotes the $\hat{k} = \tilde{k}$ line from Fig. 1. The other three contours, using the exponential example of Fig. 2, represent (ρ, β) pairs for which social welfare is the same under a RJV as under independent research (with social welfare being greater under a RJV to the left of the line and greater under independent research to the right). As the cost-effectiveness of research increases, the region for which a RJV leads to greater welfare compared to independent research expands.

region over which total surplus is greater under a RJV than under independent research expands as the effectiveness of R&D output in reducing the unit cost of production improves (i.e. as λ increases).

5. Conclusion

Do RJVs lead to greater technological improvement, profits and welfare relative to independent research? We focus on non-cooperative (competitive) RJVs and address this question in the framework of a simple two stage symmetric duopoly game allowing for inter-firm complementarity of R&D resources in the R&D production technology. We have three main conclusions.

First, if the degree of complementarity is high enough then, despite the typical free riding problem, competitive RJVs secure higher technological improvement, profits and social welfare. In this case, public policy makers promoting RJVs stand on firm economic grounds and do not need to be bothered by such issues as the extent of spillovers (appropriability of research) and whether the participants of a RJV behave co-operatively or not.

Second, if the degree of complementarity is very low then competitive RJVs lead to lower technological output relative to independent research. Furthermore, if complementarity is extremely low, we have seen in some examples that RJVs can further lead to lower profits and social welfare as well. In such situations, there is a world of difference between competitive and cooperative RJVs so far as desirable public policy is concerned. In so far as public authorities cannot diagnose whether a joint venture is organized as a co-operative venture or not, a policy of encouraging RJVs in general might be ill advised. It is necessary, therefore, to investigate the specific nature of the research project and the possibility of eliciting (binding) commitments on the resources to be shared before promoting it.

Finally, if the degree of complementarity is moderate (not too high or too low), competitive RJVs lead to higher technological improvement, profits and social welfare when spillovers are *low*. If spillovers are high, independent R&D performs better in terms of technological improvement though RJVs are still likely to lead to higher social welfare. The conventional wisdom about promoting RJVs as being a particularly good idea when spillovers are high and less of a good idea when spillovers are small must be made to stand on its head when we consider competitive RJVs and inter-firm complementarity in research. Policies that blindly encourage all RJVs as a means of internalizing spillovers when spillovers are high without investigating the nature of the RJV (competitive or cooperative) need to be seriously questioned.

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Appendix A. Existence and uniqueness of equilibrium

In the case of a RJV, $\partial k/\partial x_i = [1 + (x_j/x_i)^{\rho}]^{(1-\rho)/\rho}$. The best response function for x_i given x_j is solved by setting $\partial \pi_i(x_1, x_2)/\partial x_i = 0$. Thus, by differentiating (11) with respect to x_i, the best response function satisfies:

$$\frac{2}{9}[a - c + f(k(x_1, x_2))]\frac{\partial f(k)}{\partial k}\frac{\partial k(x_1, x_2)}{\partial x_i} = 1. \tag{A.1}$$

Now impose symmetry, and let $\hat{x}$ be the (candidate) equilibrium symmetric level of investment and $\hat{m} = 2^{1/\rho}$, so that the symmetric level of knowledge created is $\hat{k} = \hat{m}\hat{x}$. Observe that the LHS of (A.1) evaluated at $x_1 = x_2 = x$ converges to 0 as $x \to \infty$ (because $f'(k) \to 0$ as $k \to \infty$) and exceeds 1 as $x \to 0$ if $f'(0) > 9/[2\hat{m}^{1-\rho}(a - c)]$ which, in turn, is ensured by (9) as $\hat{m}^{1-\rho} \geqslant 1$. Therefore an equilibrium $\hat{x}$ exists. Uniqueness of $\hat{x}$ then follows directly from assumption (*). Simplifying (A.1) and rearranging terms yields:

$$G(\hat{k}) = 9/(2\hat{m}^{1-\rho}).$$

Under independent R&D, we have:

$$\frac{\partial k_i}{\partial x_i} = \left[1 + \left(\frac{\beta x_j}{x_i}\right)^{\rho}\right]^{\frac{1-\rho}{\rho}} \quad \text{and} \quad \frac{\partial k_i}{\partial x_j} = \beta\left[1 + \left(\frac{x_i}{\beta x_j}\right)^{\rho}\right]^{\frac{1-\rho}{\rho}}.$$

The best response function for x_i given x_j is solved by setting $\partial \pi_i(x_1, x_2)/\partial x_i = 0$. Thus, by differentiating (17) with respect to x_i, the best response function satisfies:

$$2\left(\frac{a - c + 2f(k_i) - f(k_j)}{3}\right)\left[\frac{2}{3}\frac{\partial f(k_i)}{\partial k_i}\frac{\partial k_i}{\partial x_i} - \frac{1}{3}\frac{\partial f(k_j)}{\partial k_j}\frac{\partial k_j}{\partial x_i}\right] = 1. \qquad \text{(A.2)}$$

Now impose symmetry, and let $\tilde{x}$ be the candidate equilibrium symmetric level of investment and $\tilde{m} = (1 + \beta^{\rho})^{1/\rho}$, so that the symmetric level of knowledge created is $\tilde{k} = \tilde{m}\tilde{x}$. Observe that the LHS of (A.2) evaluated at $x_1 = x_2 = x$ converges to 0 as $x \to \infty$ (because $f'(k) \to 0$ as $k \to \infty$) and exceeds 1 for x close to 0 if $f'(0) > 9/[2(2 - \beta^{\rho})\,\tilde{m}^{1-\rho}(a - c)]$ which, in turn, follows from (9) as $\tilde{m}^{1-\rho} \geqslant 1$ and $(2 - \beta^{\rho}) \geqslant 1$. Therefore an equilibrium $\tilde{x}$ exists, and uniqueness follows again from assumption (*). Simplifying (A.2) yields the equilibrium condition:

$$G(\tilde{k}) = \frac{9}{2(2 - \beta^{\rho})\tilde{m}^{1-\rho}}.$$

Appendix B. Proofs of propositions

Proposition 1. *$\hat{k} \geqslant \tilde{k}$ if and only if $C \leqslant 1$ where $C = (2 - \beta^{\rho})(\tilde{m}/\hat{m})^{1-\rho}$.*

Proof. Comparing (15) and (20), the proposition follows immediately from Assumption (*). □

Proposition 2. *For $\rho \in (\frac{1}{2}, 1]$, $C > 1$ for all values of $\beta \in [0, 1)$.*

Proof. For all $\rho \in (\frac{1}{2}, 1]$,

$$\frac{1 + \beta^{\rho}}{2} < \left(\frac{1 + \beta^{\rho}}{2}\right)^{\frac{1-\rho}{\rho}} = \left(\frac{\tilde{m}}{\hat{m}}\right)^{1-\rho}.$$

The result now follows immediately as $(2-\beta^{\rho})(1+\beta^{\rho})/2 \geqslant 1$. □

Proposition 3. *For* $\rho \in (0, \frac{1}{3}]$, $C<1$ *for all values of* $\beta \in [0, 1)$.

Proof. Consider that

$$\frac{\partial C}{\partial \beta} = Z\left[(2-\beta^{\rho})\left(\frac{1-\rho}{\rho}\right) - (1+\beta^{\rho})\right] \quad \text{where}$$

$$Z = \frac{\rho\beta^{\rho-1}}{2}\left(\frac{1+\beta^{\rho}}{2}\right)^{\frac{1-2\rho}{\rho}} > 0.$$

Therefore, when $\rho \leqslant \frac{1}{3}$ and $\beta < 1$, $C<1$ as C is increasing in β in this range, and $C=1$ when $\beta=1$. □

Proposition 4. *For each* $\rho \in (\frac{1}{3}, \frac{1}{2})$, *there exists a critical spillover level* $\beta(\rho) \in (0, 1)$ *such that* $C<1$ *for all values of* $\beta \in [0, \beta(\rho))$ *and* $C>1$ *for all for all values of* $\beta \in (\beta(\rho), 1)$. *The critical spillover level* $\beta(\rho)$ *falls from* 1 *to* 0 *as* ρ, *the degree of complementarity, increases from* $\frac{1}{3}$ *to* $\frac{1}{2}$.

Proof. For $\rho \in (\frac{1}{3}, \frac{1}{2})$,

$$\left.\frac{\partial C}{\partial \beta}\right|_{\beta=1} = Z\left[\frac{1-\rho}{\rho} - 2\right] < 0 \quad \text{and} \quad \left.\frac{\partial C}{\partial \beta}\right|_{\beta=0} = Z\left[2\left(\frac{1-\rho}{\rho}\right) - 1\right] > 0.$$

Notice also that $\partial C/\partial \beta$ is unimodal in this range as solving $(2-\beta^{\rho})(1-\rho)/\rho - (1+\beta^{\rho}) = 0$ for β yields

$$\beta^*(\rho) = (2-3\rho)^{1/\rho}.$$

Thus, there is a single inflection point for each $\rho \in (\frac{1}{3}, \frac{1}{2})$, and furthermore, it follows directly that:

$$[\beta(\rho)]^{\rho} < 2-3\rho \quad \text{when } \tfrac{1}{3} < \rho < \tfrac{1}{2} \tag{B.1}$$

Simple substitution of ($\rho = \frac{1}{3}$ and $\beta=1$) and ($\rho=\frac{1}{2}$ and $\beta=0$) into C shows that $\beta(\frac{1}{3})=1$ and $\beta(\frac{1}{2})=0$. It remains to show that $\beta(\rho)$ is decreasing on $\rho \in (\frac{1}{3}, \frac{1}{2})$.

$\beta(\rho)$ is defined by:

$$(2-(\beta(\rho))^{\rho})^{\rho} = \left(\frac{1+(\beta(\rho))^{\rho}}{2}\right)^{\rho-1}. \tag{B.2}$$

We use the implicit function theorem to implicitly differentiate $\beta(\rho)$ in (B.2). After substituting (B.2) and simplifying we have

$$\frac{\mathrm{d}\beta(\rho)}{\mathrm{d}\rho}=\frac{-\beta(\rho)\ln(\beta(\rho))(2-\beta(\rho)^{\rho})^{\rho}\left(\dfrac{\rho-1}{\rho}+\dfrac{1}{2}\right)\ln\left(\dfrac{1+\beta(\rho)^{\rho}}{2}\right)}{\rho^{2}(2-\beta(\rho)^{\rho})^{\rho-1}-\dfrac{\rho(1-\rho)}{2}\left(\dfrac{1+\beta(\rho)^{\rho}}{2}\right)^{\rho-2}}. \tag{B.3}$$

For $\frac{1}{3}<\rho<\frac{1}{2}$, the numerator on the RHS of (B.3) is always strictly positive as $\beta(\rho)<1$ while it equals zero at $\rho=\frac{1}{3}$ and $\rho=\frac{1}{2}$ as $\beta(\frac{1}{3})=1$ and $\beta(\frac{1}{2})=0$. The denominator on the RHS of (B.3) is strictly negative on $\rho\in[\frac{1}{3},\ \frac{1}{2}]$ if:

$$(2-\beta^{\rho})^{\rho}\left(\frac{1+\beta^{\rho}}{2}\right)^{1-\rho}<\frac{1-\rho}{2\rho}(2-\beta^{\rho})\left(\frac{2}{1+\beta^{\rho}}\right). \tag{B.4}$$

The LHS of (B.4) equals 1 by (B.2), and the RHS is greater than 1 by (B.1). This completes the proof. □

Proposition 5. *Whenever a competitive RJV leads to greater technological improvement than independent R&D, it also yields higher firm profits and generates greater social welfare.*

Proof. It is obvious that when a RJV generates greater profit and technological improvement, it also generates greater welfare. So, it is sufficient to show that profits are higher in the relevant region of the $(\beta,\ \rho)$-space where $\hat{k}>\tilde{k}$ $(C<1)$ which is given by:

$$\{(\beta,\rho)\colon 0\leqslant\beta<1 \text{ and } 0<\rho\leqslant\tfrac{1}{3}\}\cup\{(\beta,\rho)\colon 0\leqslant\beta<\beta(\rho) \text{ and } \tfrac{1}{3}<\rho\leqslant\tfrac{1}{2}\}$$

Observe that: (i) $\hat{\pi}=\tilde{\pi}(\beta)$ whenever $\beta=1$ as RJVs and independent research are identical in this case; and (ii) for $\frac{1}{3}<\rho\leqslant\frac{1}{2}$, $\hat{\pi}>\tilde{\pi}(\beta(\rho))$ as $\hat{k}=\tilde{k}(\beta)$ at $\beta=\beta(\rho)$ and the cost of attaining any given level of R&D output is lower under a RJV. Therefore, it suffices to show that:

1. for any $\rho\in(0,\ \frac{1}{3}]$, $\mathrm{d}\tilde{\pi}/\mathrm{d}\beta>0$ for all $\beta\in[0,\ 1)$ so that $\tilde{\pi}(\beta)<\tilde{\pi}(1)=\hat{\pi}$
2. for any $\rho\in(\frac{1}{3},\ \frac{1}{2})$, $\mathrm{d}\tilde{\pi}/\mathrm{d}\beta>0$ for all $\beta\in[0,\ \beta(\rho))$ so that $\tilde{\pi}(\beta)<\tilde{\pi}(\beta(\rho))<\hat{\pi}$

Now:

$$\frac{\mathrm{d}\tilde{\pi}(\beta)}{\mathrm{d}\beta}=\left(\left.\frac{\partial\tilde{\pi}}{\partial x_i}\right|_{x_1=x_2=\tilde{x}}+\left.\frac{\partial\tilde{\pi}}{\partial x_j}\right|_{x_1=x_2=\tilde{x}}\right)\frac{\partial\tilde{x}}{\partial\beta}+\left.\frac{\partial\tilde{\pi}}{\partial\beta}\right|_{x_1=x_2=\tilde{x}}.$$

From (13), $\partial\tilde{\pi}/\partial x_i=0$. It is straightforward to show that:

$$\left.\frac{\partial\tilde{\pi}}{\partial x_j}\right|_{x_1=x_2=\tilde{x}}=\frac{2}{9}G(\tilde{k})\,\tilde{m}^{1-\rho}\beta^{\rho}\quad\text{and}\quad\left.\frac{\partial\tilde{\pi}}{\partial\beta}\right|_{x_1=x_2=\tilde{x}}=\frac{2}{9}G(\tilde{k})\,\tilde{m}^{1-\rho}\beta^{\rho-1}\tilde{x}.$$

Thus:

$$\frac{d\tilde{\pi}}{d\beta} = \frac{2}{9}G(\tilde{k})\,\tilde{m}^{1-\rho}\beta^{\rho-1}\left[\tilde{x} + \beta\,\frac{\partial\tilde{x}}{\partial\beta}\right]$$

which is positive if $\tilde{x} + \beta(\partial\tilde{x}/\partial\beta) > 0$. Recalling that $\tilde{x} = \tilde{k}(1+\beta^{\rho})^{-1/\rho}$, we have:

$$\frac{\partial\tilde{x}}{\partial\beta} = -(1+\beta^{\rho})^{-\left(\frac{1+\rho}{\rho}\right)}\beta^{\rho-1}\tilde{k} + (1+\beta^{\rho})^{-1/\rho}\,\frac{\partial\tilde{k}}{\partial\beta}.$$

Thus:

$$\tilde{x} + \beta\,\frac{\partial\tilde{x}}{\partial\beta} = (1+\beta^{\rho})^{-\left(\frac{1+\rho}{\rho}\right)}\tilde{k} + \beta(1+\beta^{\rho})^{-1/\rho}\,\frac{\partial\tilde{k}}{\partial\beta}$$

which is positive if $\partial\tilde{k}/\partial\beta > 0$. Recall that Eq. (20) defines $\tilde{k}$, and rearranging this yields:

$$(2-\beta^{\rho})\left(\frac{1+\beta^{\rho}}{2}\right)^{\frac{1-\rho}{\rho}} = \frac{9}{2G(\tilde{k})}. \tag{B.5}$$

The RHS of (B.5) is increasing in $\tilde{k}$ as $G(\tilde{k})$ is decreasing in $\tilde{k}$. Thus, if the LHS is increasing in β, then $\partial\tilde{k}/\partial\beta > 0$ as required. Differentiating the left hand side of (B.5) with respect to β and rearranging terms gives:

$$(1-\rho)(1+\beta^{\rho})^{\frac{1-\rho}{\rho}}\,\beta^{\rho-1}\left(\frac{2-\beta^{\rho}}{1+\beta^{\rho}} - \frac{\rho}{1-\rho}\right)$$

which is positive as in the region of the (β, ρ)-space where $\hat{k} > \tilde{k}$

$$\beta^{\rho} < 2 - 3\rho.$$

To see the last inequality observe that $2-3\rho \geqslant 1$ for $0<\rho\leqslant\frac{1}{3}$ so that the inequality holds for all $\beta\in[0,1)$. Further, for $\frac{1}{3}<\rho<\frac{1}{2}$ and $\beta\in[0, \bar{\beta}(\rho))$, $\beta^{\rho} < [\bar{\beta}(\rho)]^{\rho} < 2-3\rho$ using (B.1). This completes the proof. □

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