CORRIGENDUM TO "ON THE MAXIMUM NUMBER OF CONSECUTIVE INTEGERS ON WHICH A CHARACTER IS CONSTANT"

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Theorem 1 in [1] should be corrected to the following:

Theorem 1. *If χ is any non-principal Dirichlet character to the prime modulus p which is constant on $(N, N+H]$, then*

$$H < \left\{ \frac{\pi}{2}\sqrt{\frac{e}{3}} + o(1) \right\} p^{1/4} \log p,$$

where the $o(1)$ terms depends only on p. Furthermore,

$$H \leq \begin{cases} 3.38 p^{1/4} \log p, & \text{for all odd } p, \\ 1.55 p^{1/4} \log p, & \text{for } p \geq 10^{13}. \end{cases}$$

There are two differences:

(1) The explicit constant for all p is changed from 3.64 to 3.38 (an improvement).
(2) The bound $1.55p^{1/4} \log p$ is proven for $p \geq 10^{13}$ instead of $p \geq 2.5 \cdot 10^9$.

To prove that $H(p) \leq 1.55p^{1/4} \log p$, the only changes in the proof involve correcting Table 2 in [1]. To correct it, replace the first three rows of Table 2 of [1] with the following two rows:

w	h	p	w	h	p	w	h	p
8	36	$[10^{13}, 10^{13.36}]$	8	40	$[10^{13.36}, 10^{13.5}]$	8	41	$[10^{13.5}, 10^{14.4}]$
8	44	$[10^{14.4}, 10^{14.9}]$	9	45	$[10^{14.9}, 10^{16}]$	9	51	$[10^{16}, 10^{17}]$

TABLE 1. These two rows would replace the first three rows of Table 2 in [1]. The mistake in [1] stemmed from coding incorrectly the function $\gamma(p, w, h)$.

With respect to the error involving the bound for all p. In [1] we chose "large" h to circumvent the constraint $(h/2)^{2/3}p^{1/3} \geq H$, however, this constraint is illusory. If H is larger, we can pick a $H' \leq H$ such that $H' \leq (h/2)^{2/3}p^{1/3}$ and then use Proposition 1 on H'. It turns out that the choices of h in [1] were not valid because there is a factor $g(x)$ in the calculation of $\gamma(p, w, h)$ which was miscoded and this factor can be negative when h is large with respect to p. But smaller h's would avoid this problem and they come at no penalty because the constraint $(h/2)^{2/3}p^{1/3} \geq H$ is irrelevant. We make the following changes to correct the proof and, in the process, get an improvement:

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(1) We use Brauer's inequality $H < \sqrt{2p} + 2$ for $p \leq 10^{6.1}$ as opposed to $p \leq 3 \times 10^6$ from [1]. In this narrower range it implies $H < 3.38p^{1/4} \log p$.
(2) For $p \in [10^{6.1}, 10^7]$ we choose $w = 4$ and $h = 9$. This choice of w, h satisfies all the constraints and $\gamma(p, w, h) \leq 3.38$.
(3) For $p \in [10^7, 10^{10}]$ we choose $w = 5$ and $h = 17$.
(4) For $p \in [10^{10}, 10^{13}]$ we choose $w = 6$ and $h = 28$.

We also have mistakes in Remark 2 and Remark 3 of [1]. In Remark 2 we try to prove that Norton's claims are correct, namely that for $p > e^{15}$, it is true that $H(p) < 2.5p^{1/4} \log p$. The proof once again makes the mistake of taking an h that does not satisfy the constraints. What we can prove is that $H(p) < 3p^{1/4} \log p$ for $p > e^{15}$. To correct the proof we make the following changes:

(1) Let $w = 4, h = 11$. Then in the range $p \in [e^{15}, 10^7]$ we have $\gamma(p, w, h) < 3$.
(2) For $p \in [10^7, 10^9]$ we choose $w = 5$ and $h = 16$.
(3) For $p \in [10^9, 10^{13}]$ we choose $w = 6$ and $h = 28$.

For Remark 3, we should change the bound of $3p^{1/4} \log p$ to $3.1p^{1/4} \log p$ for the case of the maximum number of consecutive non-residues for which χ remains constant. The proof requires the following changes:

(1) We use Hudson's inequality: $H < p^{1/2} + 2^{2/3}p^{1/3} + 2^{1/3}p^{1/6} + 1$ for $p \leq 10^{6.4}$ as opposed to $p \leq 2 \cdot 10^6$. With this change we get that $H(p) < 3.1p^{1/4} \log p$ for $p < 10^{6.4}$.
(2) For $p \in [10^{6.4}, 10^7]$ we choose $w = 5$ and $h = 10$. Then $\gamma(p, w, h) \leq 3.1$.
(3) For $p \in [10^7, 10^9]$ we choose $w = 5$ and $h = 16$.
(4) For $p \in [10^9, 10^{13}]$ we choose $w = 6$ and $h = 36$.

References

1. Enrique Treviño, *On the maximum number of consecutive integers on which a character is constant*, Mosc. J. Comb. Number Theory **2** (2012), no. 1, 56–72. MR 2988388

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