A Writing-intensive FYS Course on Recreational Mathematics

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Abstract

In this article I discuss my writing-intensive first-year studies course, *Recreational Mathematics*. I describe in-class activities on Benford's law and Buffon's needle, the class's main writing assignments (in which students maintained a course blog about Martin Gardner's columns and other mathematical topics they found interesting), and the final poster presentation assignment.

1 Background and Context

Lake Forest College is a liberal arts institution in the greater Chicago area with around 1500 students. The college has a campus-wide first-year studies program (FIYS) with the aim of introducing students to the college and preparing them to write at a college level. The program invites faculty from all departments to submit a proposal for an FIYS course. Every first-year student must take an FIYS course. The typical FIYS course has around 15 students. The students select five FIYS courses they are interested in (out of around 25), and then the administration chooses which class to assign them to from their five choices. This means that many of the students in the FIYS class have an interest in the subject area being taught. A professor teaching an FIYS also acts as the advisor for the students in their class. Because of this extra responsibility, there is a stipend to teach the course. The FIYS course meets three times per week for 50 minutes in a 15-week semester. The course counts as one Lake Forest College credit, which is the equivalent of 4 credit hours at most US institutions.

2 Mathematical Theme

The theme of the course is *Recreational Mathematics*. I chose this topic because I think it engages students regardless of their background, and it is a topic open to writing assignments. One way the course reaches students is that we study games and we look at surprising phenomena. For examples about games, we analyze strategies in Sudoku, we explore variations of Sudoku, and we look at the frequency distribution of the properties in the game of Monopoly. Examples of surprising phenomena we cover in class are Benford's law and Buffon's needle. In Section 3, I go into detail about how I do this in the course. My goal is to showcase that mathematics is not just about calculating and doing rote computations. At the same time, I want to have students not fear mathematics and to appreciate its beauty. By showing them that many games are mathematical, they can understand that they like mathematics without even knowing it.

The theme is also good for writing assignments, which is one of the requirements for FIYS courses at Lake Forest College. The goal is for students to be able to write at a college level. I will describe the assignments I give students in Section 3. About 85% of the classes were dedicated to mathematics. The rest were dedicated to college life such as explaining how to use the library or the resources they have at the career advancement center. There was no class time dedicated to teach writing techniques, yet there was class time used to explain the assignments and my expectations. There was also one class period dedicated to discussing the summer reading assignment (discussed in Section 3.1).

3 Course Structure

The course meets three times a week for 50 minute sessions. Some sessions consist of lectures, most included in-class activities, and there were three important assignments.

As mentioned in Section 2, we cover a lot of surprising phenomena in the classroom. For the classroom activity on Buffon's needle,[1] I bring sheets of paper with equidistant parallel lines, and I bring toothpicks to throw on the sheet of paper.[2] The students then record the number of toothpicks that "hit" a line. At the end of the class, we look at how the number of hits per student are remarkably close to each other, and that the average proportion of hits is close to $2/\pi$. For the classroom activity on Benford's law,[3] I bring newspapers to the classroom, and I ask students to record how many times each digit appears as a leading digit.[4] I ask students to ignore numbers that are used for dates or ages, as those skew the data. After all of this, we look at the distribution and find out that it's not uniformly distributed. The digit 1 is more likely than the digit 2, which is more likely than the digit 3, and so on. We then dive into the mathematical reasons for this phenomena.

My main references for topics to cover in class are articles by Martin Gardner [3], puzzles from Pete Winkler [7], [8], articles by Ian Stewart [4], and articles from the Art of Mathematics website [2] (especially for good Sudoku activities).

The course grade evaluates four main areas. Class participation is worth 15%. There is a summer assignment which is worth 5% of the grade. There are three blog posts the students have to write, they are worth 10%, 15%, and 15%, respectively. Blog participation is worth 15%. Finally, there is a poster presentation at the end of the semester worth 25%.

3.1 Summer Assignment

FIYS courses at Lake Forest College include a summer assignment. The goal of the summer assignment is to have instructors engage with students before arriving on campus, since the instructor is their advisor, and to have a topic of discussion on orientation week. In my course, I assign students to read *Logicomix: An Epic Search for Truth* written by Apostolos Doxiadis and Christos H. Papadimitriou with art by Alekos Papadatos, Annie Di Donna [1]. *Logicomix* is a graphic novel that gives a biography of Bertrand Russell and explains important concepts from set theory. I chose *Logicomix* because I think it conveys hard mathematical ideas through an interesting story. The book also gives motivation for abstract thinking. Furthermore, I think the book illustrates why many of us love mathematics so much. Finally, I think the book makes students think differently about mathematics. With this in mind, the summer assignment is to write a 750 word essay that touches on the following two questions:

- What differences do you find between mathematics as described by the characters in *Logicomix* and mathematics as taught in high school?
- Did the reading make you view mathematics differently? If so, what changed?

The students have to turn in the assignment a week before classes start. Then, on the first day of class, students get feedback on their assignment, and then they revise it. We discuss *Logicomix* during a class period a few days later.

[1] Consider parallel lines in the plane equally spaced at distance d from each other. Consider throwing a needle of length $\ell \leq d$ to the plane. Buffon proved that the probability of the needle intersecting one of the lines is $\frac{2}{\pi} \cdot \frac{\ell}{d}$ [6].

[2] The toothpicks have the same length as the distance between two parallel lines.

[3] In numerical data that includes numbers from different orders of magnitude, the frequency distribution of the leading digits is not uniform. Benford's Law predicts that 1 is the leading digit with proportion $\log(2) \approx 0.301$, 2 is the leading digit with proportion $\log(3/2) \approx 0.176$, and in general i is the leading digit with proportion $\log((i+1)/i)$ for $i = 1, 2, \ldots, 9$ [5].

[4] We say that the leading digit is the leftmost digit in a number. For example, the leading digit of 75435 is 7.

3.2 Course Blog

The course has a blog.[5] I ask students to submit three blog posts during the semester. I schedule it so that each day has two new posts. For example, one year, the blog had daily student posts from early September to late November.

Each of the three blog posts is different. For the first blog post, each student has to write a summary of one of the chapters in *The Colossal Book of Mathematics* [3] emphasizing what the student thinks is most interesting in the chapter. My goal for this post is for students to read recreational mathematics and to write about it.

For the second blog post, each student has to write a more detailed essay on another chapter of *The Colossal Book of Mathematics*. For this essay, they have to read at least three of the references cited by Martin Gardner and incorporate them into their blog post. For example, I ask the student to think of what parts of the references they think should have been part of Gardner's essay. My goal for this post is for students to read sources and to think of the choices an author makes when deciding what to include in an article.

For the third blog post, the student has to write an essay on some mathematics content they discovered for themselves. I provide suggestions for students that request them, but the idea is for students to find some mathematical puzzle or mathematical idea that they find fascinating and want to write about. My goal for this post is for students to be active in choosing what they want to write about and for me to see what mathematical aspects intrigue them. The prompts for the three blog posts can be found in the Appendix.

Finally, to ensure the blog has a life of its own, students are required to post one comment (or more) on at least two posts each week. The comment has to be a well-written post of at least 250 words. The blog participation grade also considers the quality of these comments.

The reason I like to use a blog in the course is that it is a way to get students to read their peers' work. Given that I frequently read blogs online, it also makes it easier for me to keep track of my students' progress during the semester.

3.3 Poster Presentation

The students are asked to give a poster presentation in groups of two or three. They can choose the same topic from the third blog post or find a new topic. They have to submit a draft first, then print the poster and present in front of an audience. I invite other faculty to attend the poster presentation. Students place their posters at different locations in a room and people can roam around as the students present their posters multiple times over the span of 45 minutes. Examples of topics of posters my students have done include "What's the best monopoly strategy?", "The Tower of Hanoi", and "The mathematics of snowflakes".

4 Reflections

I have taught *Recreational Mathematics* twice. The course structure described in Section 3 comes from the second time I taught the course. The first time I taught it, I had a final paper assignment instead of a poster presentation. The final paper assignment was to write a brief biography of a mathematician (chosen out of a list I gave them) and talk about one of their main contributions to mathematics.[6] The main reason I changed this assignment was that I wanted my students to develop presentation skills. Another motivating factor was that my favorite assignment was the third blog post, where they chose a topic and explored the mathematics surrounding it. The students enthusiasm shined through, and I think students learn more by researching

[5] The URL for the course is: http://fiys169.blogspot.com/.

[6] The prompt for the final paper is in the Appendix.

topics they are interested in. By assigning a poster presentation, I essentially made the students focus on a project more to their liking.

Changing the final paper to a poster presentation was very successful. The students did a better job with the assignment. They practiced a different skill (presenting), and they interacted with their classmates outside the class. The students enjoyed the poster session and seemed eager to share the mathematical topic they chose with their peers. Furthermore, some posters were so well received that my colleagues encouraged me to nominate them for the College Symposium[7] the following semester. Two of the posters from the FIYS class were presented the following semester at the Symposium.

One of the challenges I felt teaching the class was that I have no training in teaching writing. Because of this issue, I worried that my assignments did not have good prompts, and I worried about whether I graded fairly. Another obstacle is that English is not my first language.[8] If I had to teach the class again, I would ask my Humanities colleagues for advice on grading rubrics and writing prompts.

I think the main strength of the course was showing students how mathematics is more deeply connected to their everyday life than they might have realized. A professor in the Education department surveyed my students (informally) about what they got from my class, and their reply was that mathematics was everywhere. I thought the lectures were usually fun for the students (or at least had them engaged). There are certainly things to improve in my lectures. For example to give a justification for Benford's law, I introduced the concept of logarithmic density.[9] This was too advanced for students with little Calculus background.

Another strength of the class was the activity in the course blog. While most of the comments were written mainly because students had to do it for their grade, the creation of daily mathematical content was something I was proud of. The blog assignment that generated more excitement among the students was the third assignment, where they were able to choose their own topic. Because students chose their own topics, the other students would get more involved with the comments on these posts. That assignment added a community-feel to the class. One way to improve future classes might be to allow students to have more choices in topics instead of assigning chapters from Martin Gardner's book [3].

References

[1] Apostolos Doxiadis, Christos H. Papadimitriou, Alekos Papadatos, Annie Di Donna, *Logicomix*, Bloomsbury, New York, 2009

[2] Volker Ecke, Christine von Renesse, Julian F. Fleron, and Phillip K. Hotchkiss, *Discovering the art of mathematics: Games and puzzles*, 2018, Art of Mathematics. Web. September 2018.

[3] Martin Gardner, *The colossal book of mathematics*, W. W. Norton & Co. Inc., New York, 2001

[4] Ian Stewart, *Math hysteria: Fun and games with mathematics*, Oxford University Press, 2004.

[5] Wikipedia contributors, "Benford's law," *Wikipedia, The Free Encyclopedia*, `https://en.wikipedia.org/wiki/Benford%27s_law` (accessed March 3, 2019).

[6] _____, "Buffon's needle problem," *Wikipedia, The Free Encyclopedia*, `https://en.wikipedia.org/wiki/Buffon%27s_needle_problem` (accessed March 3, 2019).

[7] Peter Winkler, *Mathematical puzzles: a connoisseur's collection*, A K Peters, Ltd., Natick, MA, 2004.

[7] Lake Forest College has a symposium every year, where students present to each other.

[8] My first language is Spanish.

[9] Let A be a set and $A(x) = \{a \leq x \,|\, a \in A\}$ be the counting set of A. The natural density of A is the limit of $A(x)/x$ as $x \to \infty$. The logarithmic density of A is $\lim_{x\to\infty} \frac{1}{\ln(x)} \sum_{a\in A(x)} \frac{1}{a}$. If we let A be the set of numbers with leading digit 1, the natural density of A does not exist. However, the logarithmic density does and it matches what Benford's Law predicts.

[8] _____, *Mathematical mind-benders*, A K Peters, Ltd., Wellesley, MA, 2007.

Appendix

A Course Blog Prompts

The following are the three blog post prompts assigned throughout the semester. Their essays were posted online at a course blog.

A.1 First Blog Post

You have to read the assigned chapter and write a post containing at least 500 words and at most 1000 words. The main purpose of this assignment is to read carefully and critically. Your blog post will consist of:

- A summary of the chapter. For this summary it would help to answer questions such as "what is the chapter about?" and "what is the main concept or idea the author is communicating in the chapter?".
- Your thoughts on the chapter. Some motivating questions are "was the chapter interesting?" and "was the chapter surprising?". You should also include questions the chapter raised for you.

Your assignment also includes reading the chapters the other students write about and comment on at least four blog posts. To insure that you do not leave all comments to the last minute, you should post each comment on different days. This means the day you post a comment is different for each comment you make. The comments you write should be written seriously and contribute to the discussion. The comment has to be a well-written post of at least 200 words. Your blog participation grade will depend on the quality of these comments. When commenting on a post about a chapter in the book, you need to read the chapter yourself as well. An example of a good comment would be a question the reading raised for you. Another example would be a comment describing what you think about the reading or a comment regarding whether you agree with the students blog post.

A.2 Second Blog Post

You have to read the assigned chapter and three references cited at the end of the chapter. Your blog post will consist of:

- A brief summary of the chapter. For this summary it would help to answer questions such as "what is the chapter about?" and "what is the main concept or idea the author is communicating in the chapter?".
- For each of the three references you picked, find something you wished the author had included in his chapter.
- For each of the three references you picked, find something you are happy the author ommited in his chapter.

Your assignment also includes reading the chapters the other students write about and comment on at least four blog posts.

A.3 Third Blog Post

You have to find a subject on your own and write about it. The assignment has the following rules:

- First, pick a mathematical subject. It can be a cool puzzle, an interesting game (that involves thinking), an application of mathematics to some field (maybe an application of math to sports) or an exciting theorem.
- Once you pick your subject, find three references to give you background on that subject.
- Send the subject and the three references to my email at least one week before your blog is due (for example, if your blog is due Nov. 12, you have to send me the information by Nov. 5). I will then approve the topic, if it is suitable. If not, you have to find one right away. I suggest, you have three subjects in case I don't approve one of them.
- Once you have my approval, write a blog post talking about the subject. You can think of how Martin Gardner wrote his essays (each chapter in the textbook) as inspiration. Your blog post should be between 800 and 1200 words. No longer and no shorter. You are welcome to include pictures and you should include bibliography.

Your assignment also includes reading the posts the other students write and comment on all of them by the last day of class. The comments you write should be written seriously and contribute to the discussion.

B Poster Presentation Prompt

The following is the prompt given to students for their poster presentation. The students receive this prompt on the first week, references are due on week 12, and they present their poster on week 14.

You will have to present a poster on the last day of class. You will work on the poster in a group of two people (I will accept groups of three in some cases, but you have to request it via email). You must find a mathematical topic of your interest (it can be the same as one of the third blog posts) and create a poster for it. You will have to be ready to present a draft (a pdf version of your poster) five days before the poster presentation is due. The printed version of the poster will be due two days before the presentation is due. On the day of the presentation, we will keep the posters up all day, but you will have to be next to your poster from 9am to 9:50am.

Furthermore, you need to do the following two weeks before the presentation is due:

- MLA style citation to a journal article you intend to use as a reference in your poster presentation. The article must come from using one of the college databases; include a permanent link to the article in the database.
- MLA style citation to a book you intend to use as a reference for your poster that is in our library or that you requested from I-share.[10]
- MLA style citation for at least three other references.

C Final Paper Prompt

The following is the prompt given to students for their final paper. The students receive this prompt on the first week, the reference list is due on week 11, the first draft on week 13, the final draft on week 15.

[10] I-Share is a catalog of over 80 Illinois academic libraries that are connected to Lake Forest College.

You must write about one of the mathematicians listed in the course website.[11] The paper should give a brief biography of the mathematician and you should talk about one of their main contributions to mathematics. For example, if you could choose Bertrand Russell (you can't because he's not in the list), then you would give a brief biography of Russell and talk about his contributions to set theory with his book "Principia Mathematica". The book you read in the summer (*Logicomix*) gave a very nice example of a way to give a biography and talk about the math contributions from a mathematician (in this case Bertrand Russell). This paper should be a little longer than the blog posts. It should be at least 1200 words and at most 2000 words (if you need more space to write about the mathematical concept, you can request an extension).

There are three deadlines associated to the final paper. The first deadline is for the annotated bibliography. For the Friday of week 11 you must have the following (and turn it in):

- MLA style citation to an article you intend to use in your final paper that you found using one of the college databases; include a permanent link to the article in the database.
- MLA style citation to a book you intend to use in your paper that is in our library or that you requested from I-Share.
- MLA style citation for at least three other references.

The second deadline is for a first draft for the paper. This draft is due one week before the final day of classes (week 13). This should have at least 1200 words and a bibliography.

The third deadline is for the final paper. After I hand you back comments on your first draft. You work on the final paper and turn it in on the day of the final exam.[12] You can turn it in before, but failure to turn it in by the final exam date will mean an automatic fail for the class.

[11] The URL was: http://campus.lakeforest.edu/trevino/Fall2014/FIYS169/ListOfMathematicians.html/.

[12] Every class in Lake Forest is assigned a day for its final exam. If a class has a final paper instead of a final exam, the paper is due on that day.