

Compound Interest

1 Compounded Annually

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded annually. Then after n years, Patricia will have $1000(1 + .03)^n$ dollars. In general if she deposits A dollars at an account that yields an i per cent interest rate compounded annually, then after n years, Patricia will have

$$A \left(1 + \frac{i/100}{1} \right)^n .$$

2 Compounded Monthly

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded monthly. Then after n years, Patricia will have $1000 \left(1 + \frac{.03}{12} \right)^{12n}$ dollars ¹. In general if she deposits A dollars at an account that yields an i per cent interest rate compounded monthly, then after n years, Patricia will have

$$A \left(1 + \frac{i/100}{12} \right)^{12n} .$$

3 Compounded Daily

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded daily. Then after n years, Patricia will have $1000 \left(1 + \frac{.03}{365} \right)^{365n}$ dollars ². In general if she deposits A dollars at an account that yields an i per cent interest rate compounded daily, then after n years, Patricia will have

$$A \left(1 + \frac{i/100}{365} \right)^{365n} .$$

¹The 12 in the formula comes from the fact that there are 12 months in a year.

²The 365 in the formula comes from the fact that there are 365 days in a year.

4 Compounded Hourly

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded daily. Then after n years, Patricia will have $1000(1 + \frac{.03}{8760})^{8760n}$ dollars ³. In general if she deposits A dollars at an account that yields an i per cent interest rate compounded hourly, then after n years, Patricia will have

$$A \left(1 + \frac{i/100}{8760} \right)^{8760n}.$$

5 Compounded in General

Suppose you break a year into m pieces of equal length. For example, breaking a year into 12 pieces means you broke it into months, breaking it into 365 pieces means you broke it into days and breaking it into 8760 pieces means you broke it into hours. A more general form of compounding is by compounding m times a year.

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded m times a year. Then after n years, Patricia will have $1000(1 + \frac{.03}{m})^{mn}$ dollars. In general if she deposits A dollars at an account that yields an i per cent interest rate compounded m times a year, then after n years, Patricia will have

$$A \left(1 + \frac{i/100}{m} \right)^{mn}.$$

6 Continuous Compounding

As m gets larger and larger the amount of money in the bank gets larger and larger, however it approaches a limit. Compounding continuously is as if one would break a year into infinitely many pieces:

Suppose Patricia deposits \$1000 dollars into an account that yields a 3% interest rate compounded continuously. Then after n years, Patricia will have $1000e^{.03n}$ dollars. In general if she deposits A dollars at an account that yields an i per cent interest rate compounded continuously, then after n years, Patricia will have

$$Ae^{.03n}.$$

³The 8760 in the formula comes from the fact that there are 8760 hours in a year.

7 Example Table

The table below illustrates the amount of money Patricia would have after depositing \$1000 into an account with 3% interest compounded in different ways (annually, monthly, daily, hourly and continuously). Note how the numbers in each row get larger and larger as they get closer and closer to the value all the way to the right.

Years	Comp. Annually	Comp. Monthly	Comp. Daily	Comp. Hourly	Comp. Continuously
1	1030.	1030.42	1030.45	1030.45	1030.45
2	1060.9	1061.76	1061.83	1061.84	1061.84
5	1159.27	1161.62	1161.83	1161.83	1161.83
10	1343.92	1349.35	1349.84	1349.86	1349.86
25	2093.78	2115.02	2116.93	2117.	2117.
50	4383.91	4473.31	4481.41	4481.68	4481.69
100	19218.6	20010.5	20083.1	20085.4	20085.5
n	$1000(1 + .03)^n$	$1000 \left(1 + \frac{.03}{12}\right)^{12n}$	$1000 \left(1 + \frac{.03}{365}\right)^{365n}$	$1000 \left(1 + \frac{.03}{8760}\right)^{8760n}$	$e \cdot 03n$

8 History

The number e which is approximately 2.71828 is called the natural number. The mathematician who showed that e was the key constant in continuous compounding was Bernoulli. His student Euler popularized it in a calculus book in the 1700s and the letter e was standardized for the number because of Euler.