Practice Exam 3 SOLUTIONS

(1) $f(x) = a^x$ and $f(2) = 5$

a) $f(0) = a^0 = \boxed{1}$

b) $f(2) = a^2 = \boxed{5}$ (since $f(2) = 5$)

c) $f(4) = a^4 = (a^2)^2 = 5^2 = \boxed{25}$

d) $f(6) = a^6 = (a^2)^3 = 5^3 = \boxed{125}$

e) $f(8) = a^8 = (a^2)^4 = 5^4 = \boxed{625}$

(2) Logarithms

a) $\log_3(1) = 0$

b) $\log_2(8) = 3$ because $2^3 = 8$

c) $\log(10^{10}) = \boxed{10}$

d) $\ln(10) = 2.30259$

e) $\ln(10^{10}) = \dfrac{\log(10^{10})}{\log(e)} = \dfrac{10}{\log(e)} = \dfrac{10}{0.434294} = 23.0259$

or $\ln(10^{10}) = 10\ln(10) = 10(2.30259) = 23.0259$

(3) a) $2^x = 128$

so $x = \log_2(128) = \dfrac{\log(128)}{\log 2} = \boxed{7}$

b) $100(1.02)^x = 256$

$(1.02)^x = 2.56$

$x = \dfrac{\log(2.56)}{\log(1.02)} = \boxed{47.4688}$

c) $x^7 = 2187$

so $x = (2187)^{1/7} = \boxed{3}$

(4) Let p be the annual interest rate in percent, by the compound formula, after 5 years Alice should have

$$1000\left(1+\frac{p}{100}\right)^5$$

but she knows she has 1503, so

$$1000\left(1+\frac{p}{100}\right)^5 = 1503$$

so $$\left(1+\frac{p}{100}\right)^5 = 1.503$$

$$1+\frac{p}{100} = (1.503)^{1/5}$$

$$p = 100(1.503)^{1/5} - 100$$
$$= 8.49$$

So $\boxed{8.49\%}$

Another way to solve it is:

Once you reach $\left(1+\frac{p}{100}\right)^5 = 1.503$

then $5\log\left(1+\frac{p}{100}\right) = \log(1.503)$

so $\log\left(1+\frac{p}{100}\right) = \frac{\log(1.503)}{5} = 0.03539$

so $1+\frac{p}{100} = 10^{0.03539} = 1.0849$

so $\frac{p}{100} = 1.0849 - 1 = 0.0849$

so $\boxed{p = 8.49}$

(5) Sorting the grades we have

71, 79, 79, 83, 84, 96

a) The median is $\frac{79+83}{2} = \boxed{81.}$

b) The mean is $\frac{71+79+79+83+84+96}{6} = \boxed{82}$

c) The population standard deviation is

$$\sqrt{\frac{(71-82)^2+(79-82)^2+(79-82)^2+(83-82)^2+(84-82)^2+(96-82)^2}{6}}$$

$$= \sqrt{\frac{340}{6}} = \boxed{7.5277}$$

(6) a) $\lim_{x\to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

b) $\lim_{x\to 2} \frac{1}{x-2} = \text{"}\frac{1}{2-2}\text{"} = \text{"}\frac{1}{0}\text{"} = \text{DNE}$

c) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} x+2 = 2+2 = 4$

d) $\lim_{x\to \infty} \frac{1}{x-2} = \text{"}\frac{1}{\infty}\text{"} = \boxed{0}.$

e) $\lim_{x\to \infty} \frac{3x^3-7x^2+18000}{2x^3-2} = \boxed{\frac{3}{2}}$