1. Prove or disprove that the Boolean expressions $x \rightarrow \neg y$ and $\neg(x \rightarrow y)$ are logically equivalent.

2. Find counterexamples to disprove the following statements:
   (a) An integer $x$ is positive if and only if $x + 1$ is positive.
   (b) An integer is a palindrome if it reads the same forwards and backwards when expressed in base 10. For example, 1331 is a palindrome. All palindromes are divisible by 11.
   (c) If $a, b$ and $c$ are positive integers then $a^{(bc)} = (a^b)^c$.
   (d) Let $A, B$ and $C$ be sets. Then $A - (B - C) = (A - B) - C$.

3. Let $a$ be an integer. Prove that if $a \geq 3$, then $a^2 > 2a + 1$.

4. Let $A, B, C$ be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

5. Prove that the following identities are true for all positive integers $n$:
   (a) $1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$.
   (b) $1 + 10 + 10^2 + \ldots + 10^n = \frac{10^{(n+1)}-1}{9}$.

6. Prove that the following inequalities are true:
   (a) $e^n > n + 7$, for $n \geq 3$.
   (b) $n^2 \geq 6n + 2$, for $n \geq 7$.

7. Prove that $\sqrt{2}$ is irrational.

8. For each of the following relations defined on the set $\{1, 2, 3\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.
   (a) $R = \{(1, 1), (2, 2), (3, 3)\}$.
   (b) $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$.
   (c) $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$.
   (d) $R = \{(1, 2), (1, 3), (2, 3), (2, 2)\}$.

9. 51 small insects are in a square of $1 \times 1$. Prove that at least three insects are inside a circle of radius $1/7$.

10. True or False:
    (a) Two sets have the same cardinality if there exists a bijection from one of them to the other.
(b) The cardinality of \( \mathbb{N} \) is the same as the cardinality of \( \mathbb{R} \).
(c) The cardinality of \( 2^\mathbb{N} \) is the same as the cardinality of \( \mathbb{R} \).
(d) The cardinality of \( (0,1) \) is the same as the cardinality of \( [0,1] \).
(e) \( f(x) = 2x - 1 \) is a bijection from \( (0,1) \) to \( (0,2) \).
(f) If \( f : A \to B \) is onto and \( g : B \to C \) is onto, then \( g \circ f \) is onto.
(g) Suppose \( |A| > |B| \), then there is no one-to-one function \( f : B \to A \).
(h) Cantor’s theorem states that there is no onto function \( f : A \to 2^A \).
(i) Suppose \( f : A \to B \) is one-to-one and \( g : B \to A \) is one-to-one. Then \( f \) is onto.
(j) \( f(x) = \tan x \) is a bijection from \( (-1,1) \) to \( \mathbb{R} \).

11. Determine if the following sets are functions and explain why or why not:
(a) \( f = \{(x, y) \mid x + y = 0\} \).
(b) \( f = \{(x, y) \mid xy = 0\} \).
(c) \( f = \{(x, y) \mid x \text{ divides } y\} \).

12. Prove or disprove whether the following functions are one-to-one:
(a) \( f = \{(x, y) \mid x + y = 0\} \).
(b) \( f : \mathbb{R} \to \mathbb{R}, f(x) = 7x - 12 \).
(c) \( f = \{(1,1), (2,3), (3,2), (4,3)\} \).

13. Let \( P \) be the poset on the set \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} defined by the “divides” relation, i.e., \( a \) is related to \( b \) if \( a|b \).
(a) Does \( P \) have a maximum? What about a minimum?
(b) Find all the maximal elements of \( P \).
(c) Find all the minimal elements of \( P \).
(d) What is the height of \( P \)?
(e) What is the width of \( P \)?
(f) Is \( P \) a linear order? Why or why not?

14. Let \( A \) be the set of all finite posets. Prove that poset isomorphism is an equivalence relation on the set \( A \). (Note: The poset \( P = (X, \leq) \) is isomorphic to the poset \( Q = (Y, \leq') \) if there exists an order-preserving bijection \( f : X \to Y \). Recall that \( f \) is order-preserving if for any \( a,b \in X, a \leq b \iff f(a) \leq' f(b) \).)

15. Let \( (X, \leq) \) be a totally ordered set (i.e., it is a linear order). Define the relation \( \leq \) on \( X \times X \) as follows. If \( (x_1, y_1) \) and \( (x_2, y_2) \) are elements of \( X \times X \), then we have \( (x_1, y_1) \leq (x_2, y_2) \) provided either (a) \( x_1 < x_2 \) or else (b) \( x_1 = x_2 \) and \( y_1 \leq y_2 \). Prove that \( (X \times X, \leq) \) is a linear order.