Practice Exam 2

1. Prove that the following identities are true for all positive integers n:
   (a) $1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$.
   (b) $1 + 10 + 10^2 + \ldots + 10^n = \frac{10^{(n+1)} - 1}{9}$.
2. Prove that the following inequalities are true:
   (a) $e^n > n + 7$, for $n \geq 3$.
   (b) $n^2 \geq 6n + 2$, for $n \geq 7$.
3. Prove by induction that the sum of the angles of a convex n-gon (with $n \geq 3$) is $180(n - 2)$ degrees.
4. For each of the following statements, write the first sentences of a proof by contradiction (you should not attempt to complete the proofs).
   (a) $\sqrt{2}$ is an irrational number.
   (b) If $a > 1$, then $a^2 > \sqrt{a}$.
   (c) For all real numbers x, $x^2 \geq 0$.
   (d) If n is a multiple of 4 then $n + 2$ is not a multiple of 4.
5. Prove that if x is a real number then $x^2 \geq 0$ (you may use that for a, b, c real numbers, if $a > b$ then ($ac > bc$ if $c > 0$ and $ac < bc$ if $c < 0$)).
6. For each of the following relations defined on the set $\{1, 2, 3\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.
   (a) $R = \{(1, 1), (2, 2), (3, 3)\}$.
   (b) $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$.
   (c) $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$.
   (d) $R = \{(1, 2), (1, 3), (2, 3), (2, 2)\}$.
7. Let R be the"is similar" relation on triangles, i.e. if A and B are triangles, then ARB if and only if the angles of triangle A are the same as the angles of triangle B.
8. For each equivalence relation below, find the requested equivalence class.
   (a) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find $[1]$.
   (b) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find $[4]$.
   (c) R is has-the-same-parents-as on the set of human beings. Find [you].
   (d) R is has-the-same-tens-digits as on the set $\{x \in \mathbb{Z} : 100 < x < 200\}$. Find $[123]$.