**Practice Exam 2**

1. Prove that the following identities are true for all positive integers $n$:
   
   (a) $1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$.
   
   (b) $1 + 10 + 10^2 + \ldots + 10^n = \frac{10^{n+1}-1}{9}$.

2. Prove that the following inequalities are true:
   
   (a) $e^n > n + 7$, for $n \geq 3$.
   
   (b) $n^2 \geq 6n + 2$, for $n \geq 7$.

3. Prove by induction that the sum of the angles of a convex $n$-gon (with $n \geq 3$) is $180(n - 2)$ degrees.

4. For each of the following statements, write the first sentences of a proof by contradiction (you should not attempt to complete the proofs).
   
   (a) $\sqrt{2}$ is an irrational number.
   
   (b) If $a > 1$, then $a^2 > \sqrt{a}$.
   
   (c) For all real numbers $x$, $x^2 \geq 0$.
   
   (d) If $n$ is a multiple of 4 then $n + 2$ is not a multiple of 4.

5. Prove that if $x$ is a real number then $x^2 \geq 0$ (you may use that for $a, b, c$ real numbers, if $a > b$ then $(ac > bc$ if $c > 0$ and $ac < bc$ if $c < 0$)).

6. For each of the following relations defined on the set $\{1, 2, 3\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.
   
   (a) $R = \{(1, 1), (2, 2), (3, 3)\}$.
   
   (b) $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$.
   
   (c) $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$.
   
   (d) $R = \{(1, 2), (1, 3), (2, 3), (2, 2)\}$.

7. Let $R$ be the “is similar” relation on triangles, i.e. if $A$ and $B$ are triangles, then $ARB$ if and only if the angles of triangle $A$ are the same as the angles of triangle $B$.

8. For each equivalence relation below, find the requested equivalence class.
   
   (a) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find [1].
   
   (b) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find [4].
   
   (c) $R$ is has-the-same-parents-as on the set of human beings. Find [you].
   
   (d) $R$ is has-the-same-tens-digits as on the set $\{x \in \mathbb{Z} : 100 < x < 200\}$. Find [123].