# Practice Exam 3 Solutions

1. $\binom{20}{2}$.

2. Because
$$\sum_{k=0}^{n} \binom{n}{k} = 2^n,$$
for any $n$, then in particular, the sum
$$\binom{2n}{0} + \binom{2n}{1} + \binom{2n}{2} + \ldots + \binom{2n}{2n} = 2^{2n} = 4^n.$$
Since each binomial coefficient is positive, any one term in the sum is smaller than the sum of all the terms, hence
$$\binom{2n}{n} < 4^n.$$

3. There was a mistake in the writing of exercise 3, so please ignore the question.

4. Divide the $1 \times 1$ square into 25 equal squares of side-length $1/5$. Since there are 51 insects and 25 regions, at least three insects must be in one square of side length $1/5$. Consider the center of this square. Now from the center to the corner you can build a right triangle with side lengths $1/10$ and $1/10$ and hypotenuse equal to the distance from the center to the corner. By the Pythagorean theorem, this distance is
$$\sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2} = \frac{\sqrt{2}}{10}.$$
Therefore the three insects are contained in a circle of radius $\sqrt{2}/10$. To finish the problem we need only prove that $\sqrt{2}/10 < 1/7$. We can square and check if $2/100 < 1/49$. But this is clearly true because $98 < 100$. Therefore $\sqrt{2}/10 < 1/7$. So these three insects which are inside a circle of radius $\sqrt{2}/10$ lie also inside a circle with radius $1/7$.