Isomorphisms Worksheet

Enrique Treviño

October 21, 2014

In this series of exercises we will classify all groups of order $2p$, where p is an odd prime.

1. Assume G is a group of order $2p$, where p is an odd prime. If $a \in G$, show that a must have order 1, 2, p, or $2p$.
2. Suppose that G has an element of order $2p$. Prove that G isomorphic to $\mathbb{Z}_{2p}$.

   From now on, suppose G is not cyclic:
3. Show that G must contain an element of order p. *Hint*: Assume that G does not contain an element of order p.
4. Let z be an element of order p. Let $P = \langle z \rangle$. Show that if $g \notin P$, then g has order 2.
5. Let P be a subgroup of G with order p and $y \in G$ has order 2. Show that $yP = Py$.

   From now on, let $z \in G$ be an element of order p and $y \in G$ be an element of order 2.
6. Let $P = \langle z \rangle$ is a subgroup of order p generated by z. If y is an element of order 2, then $yz = z^{p-1}y = z^{-1}y$.
7. Prove that G is not abelian.
8. Show that we can list the elements of G as $\{y^i z^j \mid 0 \leq i \leq 1, 0 \leq j \leq p-1\}$.
9. Prove that G is isomorphic to the dihedral group D_p.