Sample Exam I Questions

1. Let $\mathbf{a} = \langle 3, -2, 7\rangle$ and $\mathbf{b} = \langle -1, 1, 1\rangle$. Calculate:
(a) $\mathbf{a}+\mathbf{b}$ (b) $\|\mathbf{a}\| + \|\mathbf{b}\|$ (c) $\mathbf{a}\cdot\mathbf{b}$ (d) $\mathbf{a}\times\mathbf{b}$

2. Let $\mathbf{a} = \langle 2, 0, 1\rangle$ and $\mathbf{b} = \langle -3, 1, 0\rangle$.
(a) Calculate $\mathbf{a} - 3\mathbf{b}$.
(b) Find a unit vector in the direction of $\mathbf{a}$.
(c) Calculate the angle between $\mathbf{a}$ and $\mathbf{b}$ to the nearest degree.
(d) Calculate the scalar component of $\mathbf{a}$ onto $\mathbf{b}$. (scalar projection)

3. Let $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{k}$.
(a) Calculate $(-\mathbf{a} + 2\mathbf{b})\cdot\mathbf{b}$.
(b) Calculate the angle between $\mathbf{a}$ and $\mathbf{b}$.
(c) Find a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

4. Find the area of the triangle formed by the vertices $(1, -1, 0)$, $(3, 4, -1)$ and $(-1, -1, 2)$.

5. Find the volume of the parallelepiped determined by the three vectors $\mathbf{a} = \langle 2, 3, -2\rangle$, $\mathbf{b} = \langle 1, -1, 0\rangle$, and $\mathbf{c} = \langle 0, 2, 3\rangle$.

6. Find an equation of the line through the point $(1, 2, 3)$ in the direction of $\mathbf{i} - \mathbf{k}$.

7. Find an equation of the line perpendicular to the plane $3x - 2z = 1$ going through the point $(1, 2, 3)$.

8. Find an equation of the plane perpendicular to the line $x = t - 1, y = -t - 3, z = 7$ and passing through the point $(1, 2, 3)$.

9. Find an equation of the plane containing both the lines $x = t - 1, y = -t + 4, z = 3$ and $x = 3t + 1, y = t + 2, z = -t + 3$.

10. Find an equation of the line that is perpendicular to both the lines in problem 9 above and passes through the point $(1, 2, 3)$.

11. Find an equation of the line that passes through the point $(1, 2, 3)$ and is parallel to both the planes $x - y = 0$ and $3x + y - z = 2$.

12. Find an equation of the plane through the point $(1, 2, 3)$ and is parallel to the line $x = -t,\ y = t + 1,\ z = 2$ and perpendicular to the plane $3x + y - z = 2$.

13. Find an equation of the plane through the three points $(1, 2, 3)$, $(4, 5, 6)$, and $(-2, 0, 4)$.

14. Given the following curves, find the position, velocity, speed, and acceleration at $t = 1$.
(a) $\mathbf{r}(t) = \cos t\,\mathbf{i} + (2 + \sin t)\mathbf{j} + (4t + 3)\mathbf{k}$ (b) $\mathbf{r}(t) = \langle t^2 - 3, t^3 - 6, 7 - t^2\rangle$

15. Given the curves defined in #14 above, find the tangent line to the point $(1, 2, 3)$.

16. For each curve in #14 above, find the arclength from $t = 0$ to $t = 1$.

17. Given an object that accelerates according to the following acceleration function, with given initial conditions, find the position of the object at time $t = 2$.
(a) $\mathbf{a}(t) = \langle 12t, -12t^2\rangle$, $\mathbf{r}(0) = \langle 1, 2\rangle$, $\mathbf{v}(0) = \langle 0, 6\rangle$
(b) $\mathbf{a}(t) = \langle e^{t/2}, 0, 6\rangle$, $\mathbf{r}(0) = \langle 1, 2, 3\rangle$, $\mathbf{v}(0) = \langle 0, 6, 0\rangle$

18. The graph of $z = xy^2$ matches choice...