Multivariable Calculus – D. Yuen

Solutions to Sample Exam I Questions

1. (a) $(2, -1, 8)$ (b) $\sqrt{62} + \sqrt{3}$ (c) 2 (d) $(-9, -10, 1)$

2. (a) $(11, -3, 1)$

(b) $(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$

(c) $\arccos \frac{-6}{5\sqrt{2}} \approx 2.58$ rad $\approx 148^\circ$

(d) $\frac{-6}{\sqrt{10}}$

3. (a) 20

(b) $\arccos 0 = \frac{\pi}{2} = 90^\circ$

(c) $\frac{1}{\sqrt{65}}(-6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) = -\frac{6}{\sqrt{65}}\mathbf{i} - \frac{5}{\sqrt{65}}\mathbf{j} + \frac{2}{\sqrt{65}}\mathbf{k}$

4. $\frac{1}{2}\sqrt{204} \approx 7.14143$

5. 19

6. $x = t + 1, y = 2, z = -t + 3$

7. $x = 3t + 1, y = 2, z = -2t + 3$

8. $x - y = -1$

9. $x + y + 4z = 15$

10. $x = t + 1, y = t + 2, z = 4t + 3$

11. $x = t + 1, y = t + 2, z = 4t + 3$

12. $x + y + 4z = 15$

13. $3x - 4y + z = -2$

14. (a) $\mathbf{r}(1) = \langle \cos 1, 2 + \sin 1, 7 \rangle$, $\mathbf{v}(1) = \langle -\sin 1, \cos 1, 4 \rangle$, speed$= |\mathbf{v}(1)| = \sqrt{17}$, $\mathbf{a}(1) = \langle -\cos 1, -\sin 1, 0 \rangle$.

(b) $\mathbf{r}(1) = \langle -2, -5, 6 \rangle$, $\mathbf{v}(1) = \langle 2, 3, -2 \rangle$, speed$= |\mathbf{v}(1)| = \sqrt{17}$, $\mathbf{a}(1) = \langle 2, 6, -2 \rangle$.

15. (a) tangent line (at $t = 0$) is $x = 1$, $y = t + 2$, $z = 4t + 3$. Or, as a parametric curve, $\ell(t) = \langle 1, 2, 3 \rangle + t\langle 0, 1, 4 \rangle = \langle 1, t + 2, 4t + 3 \rangle$.

(b) tangent line (at $t = 2$) is $x = 4t + 1$, $y = 12t + 2$, $z = -4t + 3$. Or, as a parametric curve, $\ell(t) = \langle 1, 2, 3 \rangle + t\langle 4, 12, -4 \rangle = \langle 4t + 1, 12t + 2, -4t + 3 \rangle$.

16. (a) $L = \int_0^1 (\sqrt{17})dt = \sqrt{17}$. (b) $L = \int_0^1 (\sqrt{8t^2 + 9t^4})dt = [\frac{2}{3}\frac{1}{18}(8 + 9t^2)^{3/2}]_0^1 = \frac{1}{27}(17^{3/2} - 8^{3/2})$

17. (a) $\mathbf{r}(t) = \langle 2t^3 + 1, -t^4 + 6t + 2 \rangle$, so $\mathbf{r}(2) = \langle 17, -2 \rangle$. (b) $\mathbf{r}(t) = \langle 4e^{t/2} - 2t - 3, 6t + 2, 3t^2 + 3 \rangle$, so $\mathbf{r}(2) = \langle 4e - 7, 14, 15 \rangle$.

18. The first one. (For example, setting $x = 1$ you get a parabola $z = y^2$ in the vertical plane where $x = 1$, and only the first graph has that feature.)

• See Lab #4 for further types of graphical questions, and also see HW 13.2 #53–58, and Quiz #4.