Sample Exam 2 Questions

1. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following.

(a) $z = x \ln y + y^2$ (b) $z = e^{x+xy}$ (c) $z = \sqrt{x^2 - y^2}$

2. Find $\dfrac{\partial w}{\partial x}, \dfrac{\partial w}{\partial y}, \dfrac{\partial w}{\partial z}$ for $w = xy + yz^2$.

3. Find the derivative matrix for (a) $f(x, y, z) = x - yz$ (b) $F(x, y) = (xy, x^2 + 2, xe^y)$.

4. Find $\dfrac{\partial^2 z}{\partial x^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$ for (a) $z = xy + x + y$ (b) $z = x^y$.

5. Find f_{xyz} and f_{xzx} for (a) $f(x, y, z) = x^3y^3z^3$ (b) $f(x, y, z) = e^{3x+4y+5z}$.

6. Does the function $u = e^{-13t} \sin 2x \cos 3y$ satisfy the partial differential equation $u_{xx} + u_{yy} = u_t$?

7. Find $\mathbf{D}f$ for (a) $f(x, y, z) = x + y^2 + z^3$ (b) $f(x) = (x, x^2, x^3)$ (c) $f(x, y) = (x + y, xy, \frac{x}{y})$.

8. Suppose $w = x^y$ and $x = s + t$, $y = s^2t^2$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ using the chain rule.

9. Suppose $u = y^2 e^{xz}$ and $y = \ln x + x^5 + 7$ and $z = 7 + \sin x^2$ Find $\frac{du}{dx}$ using the chain rule.

10. For $z = x^2 + y^2 - 4e^{x+y}$, $x = \sin t - \cos t$, $y = t^2 + 4t + 1$, find $\frac{dz}{dt}$ at $t = 0$ using the chain rule.

11. For $z = x^2 + y^2 - 4e^{x+y}$, $x = \sin(s + t) - 2s - 2t + 1$, $y = s^2 - 2t^2$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ at $(s, t) = (1, -1)$ using the chain rule.

12. For $z = x^3 - y^2$ and $x = x(t)$ such that $x(0) = 1$ and $x'(0) = 2$, and $y = y(t)$ such that $y(0) = 1$, $y'(0) = -2$, find $\frac{dz}{dt}$ at $t = 0$.

13. Find the differential of $w = x^3y^2$. If x is measured to be 1 ± 0.01 and and y measured to be 2 ± 0.02, use differentials to approximate the maximum possible error in calculating $w = 4$.

14. A box is measured to have dimensions 2 cm, 3 cm, 4 cm, all to within 0.01 cm. Use differentials to approximate the maximum possible error in using these measurements to calculate the volume.

15. Find the equation of the tangent plane to (a) $z = x - xy^2 + 1$ at the point $(2, -1, 1)$. (b) the surface $z^2 - z = x^2 - y^4$ at the point $(1, 1, 1)$ (c) the surface $\sin z + e^{x-y} + xyz = 1$ at $(1, 1, 0)$.

16. For the function $f(x, y) = \tan(xy)$ at the point $(\frac{1}{4}, \pi)$, (a) find the rate of change in the direction of $3\mathbf{i} + 4\mathbf{j}$; (b) find the direction in which $f(x, y)$ increases the fastest.

17. For the function $f(x, y, z) = \dfrac{2x + y}{z}$ at the point $(1, 1, 1)$, (a) find the rate of change in the direction of $-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$; (b) find the direction in which $f(x, y, z)$ increases the fastest.

18. A hill is in the shape given by elevation function $z = 100 - xye^{x-2}$. If a marble is placed at the point where $(x, y) = (2, 3)$, in which direction will it begin to roll?

19. Find the global maxima and minima (if any):

(a) $f(x, y) = x^4 + 2y^4 + x^2y$ (assume you know $f(x, y) \to \infty$ as $|x| \to \infty, |y| \to \infty$)

(b) $f(x, y) = (x^2 + y)e^{-x^2-y^2}$ (assume you know $f(x, y) \to 0$ as $|x| \to \infty, |y| \to \infty$)

20. Find the points on the surface $z = \dfrac{1}{\sqrt{xy}}$ that are closest to the origin. (You may assume that such a point exists with minimal distance.)

21. A cardboard box with a metal lid (top) is to have a volume of 24 ft^3. If the cardboard costs \$1 per square foot and the metal costs \$5 per square foot, what should the dimensions be so as to minimize the cost? (You may assume that such optimal dimensions exist.)

22. Use the method of Lagrange Multipliers to solve the following. (a) Find the minimum of $f(x, y) = x^2 + 4y^2$ subject to $2x + 3y = 25$. Assume you know the minimum exists. (b) Find the maximum and minimum of $f(x, y) = 2x + 3y$ subject to $x^2 + 4y^2 = 4$. Assume you know they exist. (c) Find the maximum and minimum of $f(x, y) = 2y + xy$ subject to $x^2 + y^2 = 24$. Assume you know they exist. (d) [For this part, just set up the system only.] Find the max and min of $f(x, y, z) = z^2 + xy$ subject to $x^2 + 2y^2 + 3z^2 = 1$.

23. Find all critical points, and find all local maxima, local minima, and saddle points:

(a) $f(x, y) = y\sqrt{x} - y^2 - x + 6y$ (b) $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

(c) $f(x, y) = xy + 1$ (d) $f(x, y) = xe^{-x} + y^3 - 3y$

(e) $f(x, y) = x^7 + x^5 + x^2 + e^2y$

(f) $f(x, y) = (x^2 + y)e^{-x-y}$ where you are given that:

$f_x = (2x - x^2 - y)e^{-x-y}$, $f_y = (1 - x^2 - y)e^{-x-y}$, $f_{xx} = (2 - 4x + x^2 + y)e^{-x-y}$,

$f_{xy} = (-1 - 2x + x^2 + y)e^{-x-y}$, $f_{yy} = (-2 + x^2 + y)e^{-x-y}$.