Multivariable Calculus – D. Yuen

Solutions to Sample Exam 2 Questions

1. (a) $\dfrac{\partial z}{\partial x} = \ln y$, $\dfrac{\partial z}{\partial y} = \dfrac{x}{y} + 2y$ (b) $\dfrac{\partial z}{\partial x} = (1+y)e^{x+xy}$, $\dfrac{\partial z}{\partial y} = xe^{x+xy}$ (c) $\dfrac{\partial z}{\partial x} = \dfrac{x}{\sqrt{x^2-y^2}}$, $\dfrac{\partial z}{\partial y} = \dfrac{-y}{\sqrt{x^2-y^2}}$

2. (a) $\dfrac{\partial w}{\partial x} = y$, $\dfrac{\partial w}{\partial y} = x + z^2$, $\dfrac{\partial w}{\partial z} = 2yz$.

3. (a) $\mathbf{D}f = [1 \quad -z \quad -y]$ (b) $\mathbf{D}F = \begin{bmatrix} y & x \\ 2x & 0 \\ e^y & xe^y \end{bmatrix}$

4. (a) $\dfrac{\partial^2 z}{\partial x^2} = 0$, $\dfrac{\partial^2 z}{\partial x \partial y} = 1$ (b) $\dfrac{\partial^2 z}{\partial x^2} = y(y-1)x^{y-2}$, $\dfrac{\partial^2 z}{\partial x \partial y} = x^{y-1} + yx^{y-1}\ln x$.

5. (a) $f_{xyz} = 27x^2y^2z^2$, $f_{xzx} = 18xy^3z^2$ (b) $f_{xyz} = 60e^{3x+4y+5z}$, $f_{xzx} = 45e^{3x+4y+5z}$.

6. Yes, you should get $-13e^{-13t}\sin 2x \cos 3y$ for both sides after differentiating.

7. (a) $[1 \quad 2y \quad 3z^2]$ (b) $\begin{bmatrix} 1 \\ 2x \\ 3x^2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ y & x \\ \frac{1}{y} & -\frac{x}{y^2} \end{bmatrix}$

8. $\dfrac{\partial w}{\partial t} = yx^{y-1} + 2s^2tx^y \ln x$, $\dfrac{\partial w}{\partial s} = yx^{y-1} + 2st^2x^y \ln x$.

9. $\frac{du}{dx} = zy^2e^{xz} + 2ye^{xz}(\frac{1}{x} + 5x^4) + xy^2e^{xz}2x\cos x^2$.

10. -14.

11. $\dfrac{\partial z}{\partial s} = -10$, $\dfrac{\partial z}{\partial t} = -22$.

12. 10.

13. $dw = 3x^2y^2dx + 2x^3ydy$. At $x = 1$, $y = 2$, $|dx| \le 0.01$, $|dy| \le 0.02$, we get $|dw| \le 0.20$.

14. $V = xyz$, so $dV = yzdx + xzdy + zydz$. At $x = 2, y = 3, z = 4$ with $|dx| \le 0.01$, $|dy| \le 0.01$, $|dz| \le 0.01$, we get $|dV| \le 0.26$ cm^3.

15. (a) $z = 4y + 5$ (b) $2x - 4y - z + 3 = 0$ (c) $x - y + 2z = 0$

16. (a) $\dfrac{6\pi + 2}{5}$ (b) $\langle 2\pi, \frac{1}{2} \rangle$

17. (a) $\frac{-10}{3}$ (b) $\langle 2, 1, -3 \rangle$

18. $\langle 9, 2 \rangle$

19. (a) Critical points at $(0,0)(f = 0)$ and $(\pm\frac{1}{2\sqrt{2}}, -\frac{1}{4})(f = -\frac{1}{128})$. No global max. global mins at $(\pm\frac{1}{2\sqrt{2}}, -\frac{1}{4})$. (b) Critical points at $(0, -\frac{1}{\sqrt{2}})(f \approx -.42888)$ and $(0, \frac{1}{\sqrt{2}})(f \approx .42888)$ and $(\pm\frac{1}{\sqrt{2}}, \frac{1}{2})(f \approx .47237)$. Global max at $(\pm\frac{1}{\sqrt{2}}, \frac{1}{2})$, global min at $(0, -\frac{1}{\sqrt{2}})$.

20. $(\dfrac{1}{\sqrt[4]{2}}, \dfrac{1}{\sqrt[4]{2}})$ and $(-\dfrac{1}{\sqrt[4]{2}}, -\dfrac{1}{\sqrt[4]{2}})$

21. Length 2 ft, width 2ft, and height 6 ft.

22. (a) The system $2x = 2\lambda, 8y = 3\lambda, 2x + 3y = 25$ has only one solution $(x, y) = (8, 3)$, so $f(8, 3) = 100$ must be the min. (b) The system $2 = 2x\lambda, 3 = 8y\lambda, x^2 + 4y^2 = 4$ has two solutions $(x, y) = (-8/5, -3/5)(f = -5)$ and $(x, y) = (8/5, 3/5)(f = 5)$. So $f = -5$ must be min and $f = 5$ must be max. (c) The system $y = 2x\lambda, 2 + x = 2y\lambda, x^2 + y^2 = 24$ has four solutions $(3, -\sqrt{15})(f = -5\sqrt{15} \approx -19.3649)$ [MIN], $(3, \sqrt{15})(f = 5\sqrt{15} \approx 19.3649)$ [MAX], $(-4, -2\sqrt{2})(f = 4\sqrt{2} \approx 5.6568)$, $(-4, 2\sqrt{2})(f = -4\sqrt{2} \approx -5.6568)$. (d) The system is $y = 2x\lambda,\ x = 4y\lambda,\ 2z = 6z\lambda,\ x^2 + 2y^2 + 3z^2 = 1$.

23. (a) Local max at $(4, 4)$
(b) Local min at $(0, 0)$, local max at $(-\frac{5}{3}, 0)$, saddle points at $(-1, \pm 2)$.
(c) Saddle point at $(0, 0)$.
(d) Local max at $(1, -1)$, saddle point at $(1, 1)$.
(e) No critical points
(f) Saddle point at $(\frac{1}{2}, \frac{3}{4})$.