Multivariable Calculus – D. Yuen

Sample Exam 3 Questions

1. Convert from polar to rectangular coordinates.
(a) $(0, 91\pi)$ (b) $(-3, \frac{\pi}{4})$ (c) $(2, \frac{2\pi}{3})$ (d) $(-2, -\frac{3\pi}{4})$

2. Convert from rectangular to polar coordinates.
(a) $(1, -1)$ (b) $(3, \sqrt{3})$ (c) $(0, -7)$ (d) $(-\sqrt{2}, \sqrt{6})$

3. Convert the following from rectangular coordinates to both cylindrical and spherical coordinates.
(a) $(0, 0, -3)$ (b) $(-1, 1, \sqrt{2})$ (c) $(0, -1, \sqrt{3})$ (d) $(\sqrt{3}, -1, 0)$

4. Convert the following spherical coordinates (in form (ρ, ϕ, θ)) to rectangular coordinates.
(a) $(2, 0, 0)$ (b) $(4, \frac{\pi}{2}, \pi)$ (c) $(1, \pi, 1)$ (d) $(3, \frac{3\pi}{4}, 2\pi)$

5. Evaluate the following integrals

(a) $\displaystyle\int_{-1}^{1}\int_{0}^{2}(6x^2y + x)\,dx\,dy$ (b) $\displaystyle\iint_R \frac{x}{(xy+1)^2}\,dA$ where $R = [0,1]\times[0,1]$.

(c) $\displaystyle\int_{0}^{\pi}\int_{0}^{3}(\sin x + \frac{y^2}{x+1})\,dy\,dx$ (d) $\displaystyle\iint_R \frac{2x}{(x^2+y^2)^2}\,dA$ where $R = [0,1]\times[1,\sqrt{3}]$

(e) $\displaystyle\int_{0}^{1}\int_{-x}^{x}(2y + x)\,dy\,dx$ (f) $\displaystyle\iint_D x\sin y\,dA$ where $D = \{0 \le y \le \frac{\pi}{2}, 0 \le x \le \cos y\}$

(g) $\displaystyle\iint_D xy\,dA$ where D is the triangle bounded by $y = x$, $y = 2x$, $x = 1$.

(h) $\displaystyle\iint_D (x^2+y^2)^{\frac{3}{2}}\,dA$ where D is the quarter disc bounded by $x^2 + y^2 = 1$ with $x \ge 0, y \ge 0$.

(i) $\displaystyle\iint_D x\,dA$ where D is the region bounded by half the cardioid $r = 1 + \sin\theta$ with $x \ge 0$

(j) $\displaystyle\iiint_B ze^{2x-y}\,dV$ where $B = [0,1]\times[0,2]\times[0,3]$

(k) $\displaystyle\iiint_B xz\sin(xy)\,dV$ where $B = [0,1]\times[0,2]\times[0,3]$

6. Let T be the tetrahedron which is bounded by $3x - 2y + z = 6$, $x = 0$, $y = 0$, $z = 0$. (a) Find the volume of T. (b) Set up the integral $\iiint_T xyz\,dV$.

7. Let B be the solid region between the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$. (a) Find the volume of B. (b) Set up the integral $\iiint_B xyz\,dV$.

8. A solid region W in the first octant is bounded by the planes $x + z = 7$, $y + z = 7$, $x = 0$, $y = 0$, $z = 0$. Set up the integral $\iiint_W (x + yz)\,dV$.

9. A solid R in the first octant is inside the cylinder $x^2 + y^2 = 3$, and bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y^3 + z^2 = 95$. Set up the integral $\iiint_R (x + yz)\,dV$.

10. A solid B is inside the cylinder $x^2 + y^2 = 3$, and between the planes $x + 2y + z = 0$ and $x - y - z = 40$. Set up the integral $\iiint_B (x + yz)\,dV$.

11. A solid T is part of the ball $x^2 + y^2 + z^2 \le 9$ in the first octant that is bounded by the planes $y = 0$, $z = 0$, and $y = x$. Set up the integral $\iiint_T (x + yz)\,dV$.

12. Let W be the solid inside the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$. Set up the integral $\iiint_W xyz\,dV$ in all three coordinate systems.

13. Let B be the eighth of a ball of radius 5 in the first octant. Set up the integral $\iiint_B x^2\,dV$ in all three coordinate systems.

14. Sketch the polar graphs:
(a) $r = 3 + 2\sin\theta$ (b) $r = |\theta - \pi| - \frac{\pi}{4}$ for $0 \le \theta \le 2\pi$.

15. ALL YOUR HOMEWORK PROBLEMS.