Multivariable Calculus – D. Yuen

**Solutions to Sample Exam 3 Questions**

**1.** (a) $(0,0)$ (b) $(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2})$ (c) $(-1,\sqrt{3})$ (d) $(\sqrt{2},\sqrt{2})$

**2.** (a) $(\sqrt{2}, -\frac{\pi}{4})$ (b) $(2\sqrt{3}, \frac{\pi}{6})$ (c) $(7, -\frac{\pi}{2})$ (d) $(2\sqrt{2}, \frac{2\pi}{3})$

**3.** (a) Cyl: (0, any, -3), Sph: $(3, \pi,$ any). (b) Cyl: $(\sqrt{2}, \frac{3\pi}{4}, \sqrt{2})$, Sph: $(2, \frac{\pi}{4}, \frac{3\pi}{4})$
(c) Cyl: $(1, -\frac{\pi}{2}, \sqrt{3})$, Sph: $(2, \frac{\pi}{6}, -\frac{\pi}{2})$ (d) Cyl: $(2, -\frac{\pi}{6}, 0)$, Sph: $(2, \frac{\pi}{2}, -\frac{\pi}{6})$

**4.** (a) $(0,0,2)$ (b) $(-4,0,0)$ (c) $(0,0,-1)$ (d) $(\frac{3}{\sqrt{2}}, 0, -\frac{3}{\sqrt{2}})$

**5.** (a) 4 (b) $1 - \ln 2$ [integrate $y$ first]
(c) $6 + 9\ln(\pi+1)$ (d) $1 - \frac{1}{\sqrt{3}} - \frac{\pi}{12}$ [integrate $x$ first] (e) $\frac{2}{3}$
(f) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\cos y} x \sin y \, dx \, dy = \frac{1}{6}$ (g) $\displaystyle\int_0^1 \int_x^{2x} xy \, dy \, dx = \frac{3}{8}$
(h) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^1 r^4 \, dr \, d\theta = \frac{\pi}{10}$ (i) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{1+\sin\theta} r^2 \cos\theta \, dr \, d\theta = \frac{4}{3}$
(j) $\frac{9}{4}(e^2 - 2 + e^{-2})$ (k) $\frac{9}{4}(2 - \sin 2)$ [integrate $y$ before $x$]

**6.** (a) $\displaystyle\int_0^2 \int_{\frac{3x-6}{2}}^0 \int_0^{6-3x+2y} 1 \, dz \, dy \, dx = 6$ (b) $\displaystyle\int_0^2 \int_{\frac{3x-6}{2}}^0 \int_0^{6-3x+2y} xyz \, dz \, dy \, dx$

**7.** (a) $\displaystyle\int_0^{2\pi} \int_0^1 \int_{3r^2}^{4-r^2} r \, dz \, dr \, d\theta = 2\pi$ (b) $\displaystyle\int_0^{2\pi} \int_0^1 \int_{3r^2}^{4-r^2} r^3 z \cos\theta \sin\theta \, dz \, dr \, d\theta$

**8.** $\displaystyle\int_0^7 \int_0^{7-z} \int_0^{7-z} (x+yz) \, dx \, dy \, dz$, or $\displaystyle\int_0^7 \int_0^{7-y} \int_0^{7-z} (x+yz) \, dx \, dz \, dy$, or
$\displaystyle\int_0^7 \int_0^{7-z} \int_0^{7-z} (x+yz) \, dy \, dx \, dz$, or $\displaystyle\int_0^7 \int_0^{7-x} \int_0^{7-z} (x+yz) \, dy \, dz \, dx$

**9.** $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{3}} \int_0^{\sqrt{95 - r\cos\theta - r^3 \sin^3\theta}} (r\cos\theta + zr\sin\theta) r \, dz \, dr \, d\theta$

**10.** $\displaystyle\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{r\cos\theta - r\sin\theta - 40}^{-r\cos\theta - 2r\sin\theta} (r\cos\theta + zr\sin\theta) r \, dz \, dr \, d\theta$

**11.** [use standard cylindrical or spherical coordinates] $\displaystyle\int_0^{\frac{\pi}{4}} \int_0^3 \int_0^{\sqrt{9-r^2}} (r\cos\theta + zr\sin\theta) r \, dz \, dr \, d\theta$,
or $\displaystyle\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^3 (\rho\sin\phi\cos\theta + \rho^2\cos\phi\sin\phi\sin\theta)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

**12.** $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} xyz \, dz \, dy \, dx$, $\displaystyle\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} zr^3 \cos\theta \sin\theta \, dz \, dr \, d\theta$,
$\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^2 \rho^5 \sin^3\phi \cos\phi \cos\theta \sin\theta \, d\rho \, d\phi \, d\theta$

**13.** $\displaystyle\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} x^2 \, dz \, dy \, dx$, $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^5 \int_0^{\sqrt{25-r^2}} r^3 \cos^2\theta \, dz \, dr \, d\theta$,
$\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^5 \rho^4 \sin^3\phi \cos^2\theta \, d\rho \, d\phi \, d\theta$