Induction Proof Practice

1. Prove that for any positive integer n,

$$1 + 3 + 6 + \ldots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}.$$

2. Prove that for any positive integer n,

$$e^n > n.$$

3. Prove by induction that the number of subsets of a set with n elements is 2^n.

4. Prove that every positive integer $n > 1$, has a prime divisor.

5. Evaluate the sum

$$1 \times 1000 + 2 \times 999 + 3 \times 998 + \ldots + 999 \times 2 + 1000 \times 1.$$