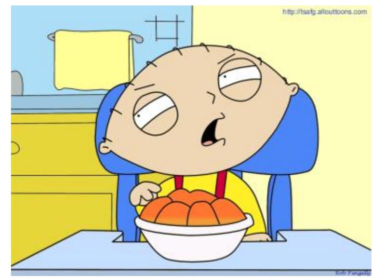
Math 210 Final Exam Name: ______________________________

Instructions: <u>Only official calculators are permitted</u>.
Show all work. Pay attention to whether a problem requires numerical evaluation or just a setup.

Question	Score	Possible
0	1	1
1		9
2		6
3		8
4		8
5		6
6		4
7		13
8		8
9		8
10		10
11		6
TOTAL		87



Math… the final frontier… These are the continuing voyages of the math class Multivariable Calculus, to explore strange new coordinate systems, to seek out vector fields and partial derivatives, to boldly triple integrate what no one has done before.

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0. (Just for fun trivia) Circle the correct words to the lyrics:

Hakuna Matata, what a wonderful

vector **derivative** **integral** **phrase**

Hakuna Matata, ain't no passing

grade **truck** **wind** **craze**

It means no

exams **labs** **studying** **worries**

for the rest of your

semester **classes** **professors** **days**

It's a problem free

Final exam **report card** **philosophy**

Hakuna Matata

1. (9 pt)

(a) Is $\mathbf{F}(x, y) = \langle 2x + 2xy, x^2 + y^2 \rangle$ a conservative vector field (a gradient vector field). If yes, find a potential function for it.

(b) Find the angle between the two vectors $\mathbf{a} = \mathbf{i} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Use your calculator to give an approximation to the nearest degree.

(c) Find a **<u>unit</u>** vector that is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ as given in part (b).

2. (6 pt)

(a) Find an equation of the tangent line to the curve $\mathbf{r}(t) = \langle t^2, 2t^4, t+4 \rangle$ at the point $(1,2,3)$.

(b) Find an equation of the tangent plane to the surface $3x^2 - 10yz - 5z^3 + z = 3$ at the point $(3,2,1)$.

3. (8pt)

(a) Suppose $w = x^y + 7z$ and $(x, y, z) = (tu, t-u, t+u)$. Use the chain rule to find $\dfrac{\partial w}{\partial u}$ at $(u,t) = (1,2)$.

(b) Use the method of Lagrange Multipliers to find the absolute minimum of $f(x, y, z) = 3x + 5y + z$ subject to the constraint $2x^2 + y^2 + z^2 = 1$. You may assume such a minimum exists.

4. (8pt)

(a) For the function $f(x, y, z) = xy + z^3$, find:

(i) ∇f

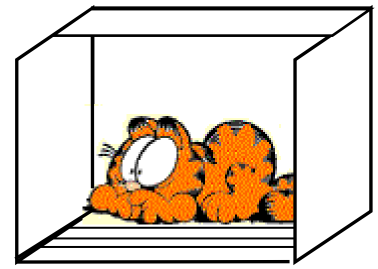
(ii) the rate of change (with respect to unit distance) of f at $(1,2,3)$ in the direction of $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. [This is just a directional derivative.]

(b) For the vector field $\mathbf{F}(x, y, z) = \langle 3y, 7y, xyz \rangle$, find $\nabla \times \mathbf{F}$ and $\nabla \cdot \mathbf{F}$.

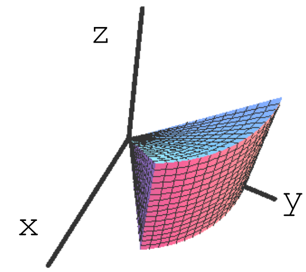
5. (6pt) Find all critical points, local maxima, local minima, and saddle points of the function

$$f(x,y) = xy^2 + x^3 - 3x \, .$$

6. (4 pt) Garfield wants to build yet another new cat box (with no front) with a carpeted bottom and where the top only goes halfway to the front. (See picture!) The carpeted bottom costs <u>twice</u> as much per area as the rest of the material per area. If the total volume of the box should be 5000 in^3, what dimensions would minimize the cost? (For this problem, you may assume such a minimum exists.)



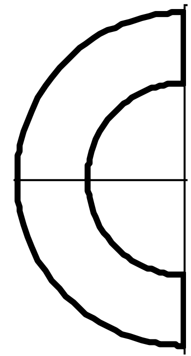
7. (13 pts) A solid W in the first octant is bounded by the cone $z=\sqrt{3x^2+3y^2}$, the cylinder $x^2+y^2=4$ and the planes $z=0$, $x=0$, and $y=\frac{x}{\sqrt{3}}$. Set up, **<u>but do not evaluate</u>**, the triple integral $\iiint\limits_W y\,dV$ using



(a) **rectangular,** (b) **cylindrical,** and (c) **spherical** coordinates.

8. (8pt)

(a) **<u>Evaluate</u>** (to a number) the double integral $\iint_D e^{x^2+y^2}\,dA$ where D is the "C" shape with $1 \le x^2 + y^2 \le 3$ with $x \le 0$. (Hint: You will need polar coordinates.)



(b) A solid E is bounded by the planes $z = 4$, $x - z = 0$, $2x - y + 3z = 4$, and the surface $x - y - z^3 = 4$. Set up, **<u>but do not evaluate</u>**, the triple integral $\iiint_E xyz\,dV$ as <u>one</u> iterated triple integral. (You should be able to do this without a 3D picture. But a 2D sketch of the base domain is always helpful.)

9. (8 pts) Consider the oriented curve C given by $\mathbf{r}(t) = \left\langle 5, e^t, t^2 \right\rangle$, with $0 \le t \le 2$. Set up, **<u>but do not evaluate</u>**, the integrals:

(a) $\int_C x^2 ds$,

(b) $\int_C z dy$,

(c) $\int_C \mathbf{F} \cdot d\mathbf{r} \; \left(= \int_C \mathbf{F} \cdot \mathbf{T} ds \right)$ where $\mathbf{F}(x, y, z) = \langle z, x, y \rangle$,

(d) an integral that gives the arclength of C.

10. (10pt)

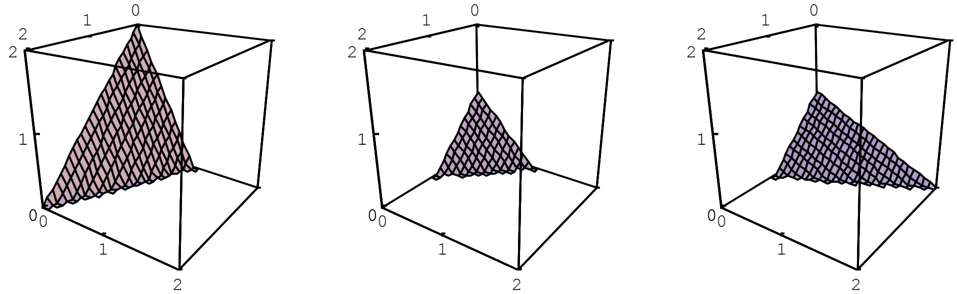
(a) An oriented surface K is given by the parametrization $\mathbf{r}(u,t) = \langle ut, u+2t, u+3t \rangle$ with domain D, where D is the triangle bounded by the lines $u=0$, $t=1$, and $u+t=3$. Set up, **but do not evaluate**, the surface integrals (a) $\iint_K \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z) = \langle x, x, y-z \rangle$ and

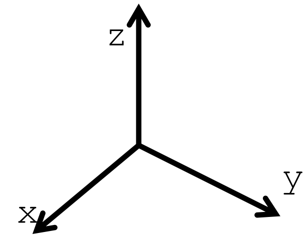
(b) $\iint_K (x^2+y^2)\,dS$

(b) Consider the surface that is the graph $z = 4 - x^2 - y^2$ with $z \geq 0$. Set up, **but do not evaluate,** an integral that calculates its surface area.

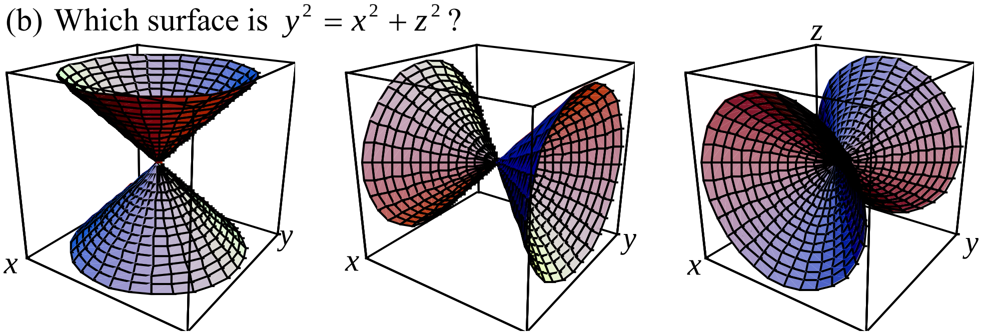
11. (6pt) Multiple choice. Just circle the correct choice. Assume all standard viewpoints.

(a) Which graph matches $2x + y + 2z = 2$?

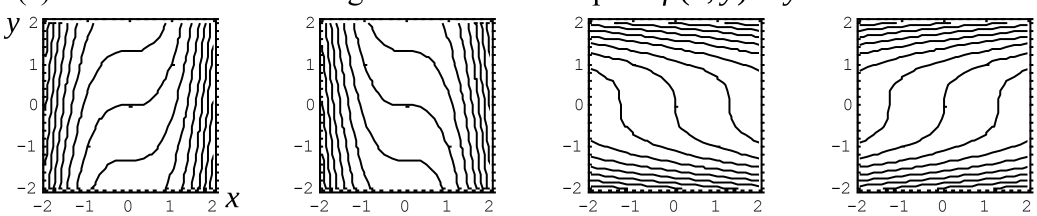


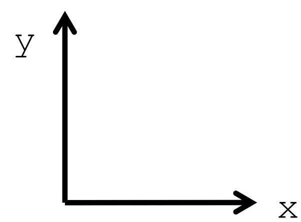


(b) Which surface is $y^2 = x^2 + z^2$?

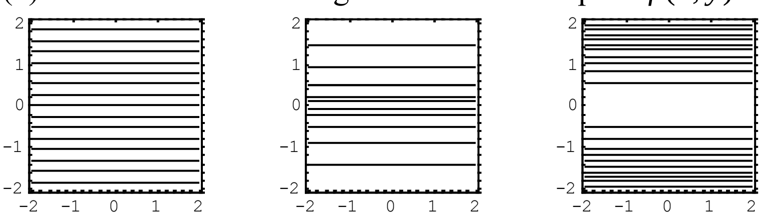


(c) Which of the following is the contour map of $f(x, y) = y + x^3$?

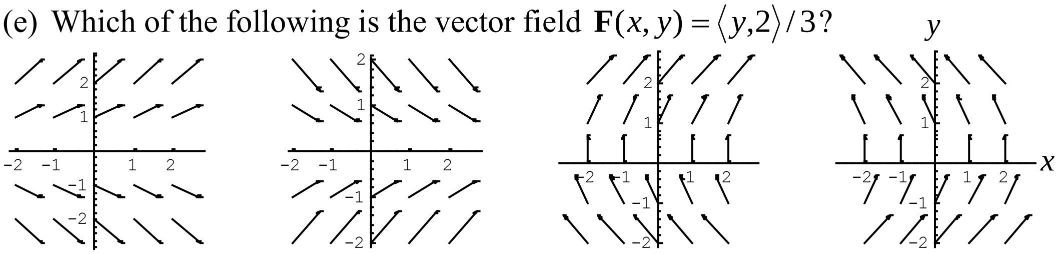




(d) Which of the following is the contour map of $f(x, y) = y^2$?



(e) Which of the following is the vector field $\mathbf{F}(x, y) = \langle y, 2\rangle / 3$?



(f) Which of the following is the vector field $\mathbf{F}(x, y) = \langle x, -y\rangle / 3$?