Homework 6

- 1. Exercises 5.7.1 and 5.7.2.
- 2. Exercises 5.7.3.
- 3. Exercises 5.8.1 and 5.8.2.
- 4. Exercise 5.8.3.
- 5. Exercises 5.8.4, 5.8.5 and 5.8.6.
- 6. We can define the cross-ratio in the plane as follows. Given four points A, B, C, D on a line, then we say that the cross-ratio [A, B; C, D] is

$$[A, B; C, D] = \left(\frac{AC}{BC}\right) / \left(\frac{AD}{BD}\right)$$

In the following diagram, prove that [A, B; C, D] = [A', B'; C', D'] (true whenever AA', BB', CC', and DD' concur):



7. Suppose we have A, B, C three points aligned. Let D be the point at infinity. Then show

$$[A, B, C, D] = \frac{AC}{BC}.$$

In other words, as D goes farther and farther away, the cross ratio approaches AC/BC.



8. Let ABC be a triangle, let M be the midpoint of AC, and let N be a point on the line BM such that AN is parallel to BC. Let P be any point on the line AC, and let Q be the intersection of the line BP with the line AN. Prove that

$$\frac{AQ}{QN} = \frac{1}{2} \left(\frac{AP}{PM} \right).$$

Hint: Use cross ratios.