## Math 340: Geometry

## Homework 6

1. Exercises 5.7.1 and 5.7.2.
2. Exercises 5.7.3.
3. Exercises 5.8.1 and 5.8.2.
4. Exercise 5.8.3.
5. Exercises 5.8.4, 5.8.5 and 5.8.6.
6. We can define the cross-ratio in the plane as follows. Given four points $A, B, C, D$ on a line, then we say that the cross-ratio $[A, B ; C, D]$ is

$$
[A, B ; C, D]=\left(\frac{A C}{B C}\right) /\left(\frac{A D}{B D}\right)
$$

In the following diagram, prove that $[A, B ; C, D]=\left[A^{\prime}, B^{\prime} ; C^{\prime}, D^{\prime}\right]$ (true whenever $A A^{\prime}, B B^{\prime}, C C^{\prime}$, and $D D^{\prime}$ concur):

7. Suppose we have $A, B, C$ three points aligned. Let $D$ be the point at infinity. Then show

$$
[A, B, C, D]=\frac{A C}{B C}
$$

In other words, as $D$ goes farther and farther away, the cross ratio approaches $A C / B C$.

8. Let $A B C$ be a triangle, let $M$ be the midpoint of $A C$, and let $N$ be a point on the line $B M$ such that $A N$ is parallel to $B C$. Let $P$ be any point on the line $A C$, and let $Q$ be the intersection of the line $B P$ with the line $A N$. Prove that

$$
\frac{A Q}{Q N}=\frac{1}{2}\left(\frac{A P}{P M}\right)
$$

Hint: Use cross ratios.

