Math 340: Geometry
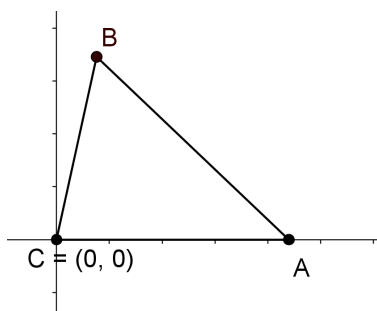## Worksheet 1: Coordinate Geometry

Figure 1: $\triangle ABC$ with C at the origin and A at the x-axis

1. Let $\triangle ABC$ be a triangle with $AB = c, AC = b, BC = a$, and $\angle ACB = \theta$. Find the coordinates (in terms of a, b, θ) of A and B in Figure 1.

2. Let $\triangle ABC$ be a triangle satisfying $AB = c, BC = a, AC = b$, and $\angle ACB = \theta$. Prove the Law of Cosines, i.e., that

$$c^2 = a^2 + b^2 - 2ab\cos(\theta).$$

   Hint: Use the coordinates from question 1.

3. Show that the altitudes of a triangle concur. Hint: Consider the triangle from Figure 1. Let D be on AC such that $BD \perp AC$. Let E be on AB such that $CE \perp AB$. Let H be the intersection of CE and BD. Find the coordinates of H and prove that $AH \perp BC$.

4. Find the coordinates (in terms of a, b, θ) of the orthocenter H of the triangle in Figure 1. Hint: It's the same as H in the previous question.

5. Find the coordinates (in terms of a, b, θ) of the centroid G of the triangle in Figure 1.

6. Find the coordinates (in terms of a, b, θ) of the circumcenter O of the triangle in Figure 1. Hint: Use that O is the intersection of the perpendicular line bisectors of the sides. You need only intersect two of them. The perpendicular line bisector of AC has a very simple formula.

7. Using the coordinates of H, G, O from previous questions, find the slopes of OG and OH. Prove that they are equal and conclude that O, G, H are aligned.

8. Prove that the length of OH is three times the length of OG to conclude that O, G, H are aligned in that order with a ratio of $1 : 2$. This is the celebrated theorem known as the Euler line.