

Homework: Chapter 2

1. Suppose someone tells you that two sides of a right triangle have lengths 5 and 7. What are all the possible lengths of the 3rd side?

The rest of the homework pertains to the equivalence of Euclid's parallel postulate to other postulates. For this homework, you may only use the following axioms and theorems, if needed.

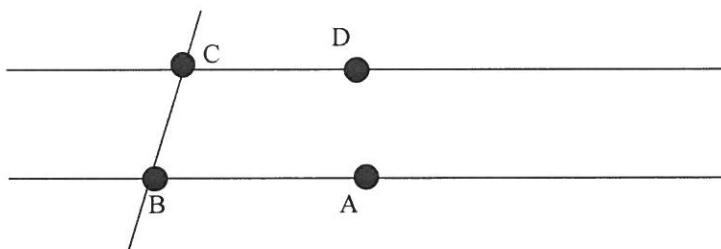
- Through any two distinct points is a unique line.
- SAS, ASA, SAA, SSS congruent triangle theorems.
- You can always drop a perpendicular from any point to a line.
- (**) If a transversal cuts two lines and if the corresponding angles are the same, then the two lines are parallel.

In particular, do **NOT** use the following

- That the sum of the angles in a triangle is 180 degrees.
- That the sum of the angles in a quadrilateral add to 360 degrees.
- That if a transversal cuts two parallel lines, then the corresponding angles are the same.
- That if a transversal cuts two parallel lines, then the interior angles add to 180 degrees.

Here are the statements of the various postulates. Remember, these are **not** to be used as theorems in this homework for #2, 3, 4, 5, 6. (But if Euclid's Fifth is assumed, then the above 4 are true.)

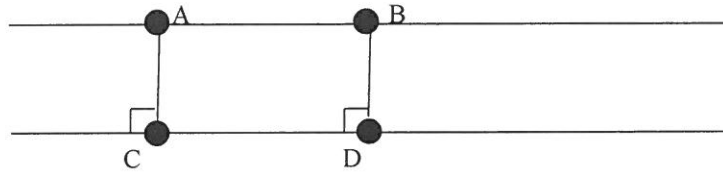
(X1) Euclid's Fifth: If a transverse cuts two lines such that the interior angles on one side add up to less than 180 degrees, then those two lines must intersect on that side. That is, if $\angle ABC + \angle BCD < 180^\circ$, then $\overrightarrow{BA} \cap \overrightarrow{CD} \neq \emptyset$.



(X2) Proclus's Postulate: If a third distinct line intersects one of two parallel lines, then it must intersect the other line.

(X3) Playfair Postulate: Given a line and a point not on the line, there is a unique line through that point parallel to the given line.

(X4) Equistant Postulate: Given two parallel lines and points A, B on one of the lines, if we drop perpendiculars to the other line, obtaining points C, D , then we must have $AC = BD$.



(X5) Triangle Angle Sum Postulate: The angles of a triangle always sum to 180° .

Exercises:

2. Prove that Euclid's Fifth implies Playfair's Postulate. Many ways to do this problem. Direct proof or proof by contradiction should work. Hint: Draw a transversal through the point.
3. Prove that Playfair's Postulate implies Euclid's Fifth. Hint: Prove the contrapositive or do a proof by contradiction. Hint: (***) may help.
4. Prove that Playfair's Postulate implies Proclus's Postulate. Prove the contrapositive or do a proof by contradiction.
5. Prove that the Equistant Postulate implies Proclus's Postulate. Hint: do a proof by contradiction. Hint: do strategic perpendicular drops.
6. Prove that Playfair's Postulate implies the Equidistant Postulate. Many ways to do this one. Direct proof or proof by contradiction should work.

Disclaimer: The above hints are meant to be one way to solve the problem but not the only way.