Senior Seminar
Homework for Chapter 3

1. (a) Given positive integers $A > B$, and positive integer m, prove that the integer $(A^m - B^m)$ is a multiple of $(A - B)$.

   (b) Let $k > 1$ be a positive integer. Prove that if $2^k - 1$ is a prime number, then k must be prime. Hint: proof by contradiction.

2. (a) Given positive integers A, B, and odd positive integer m, prove that the integer $(A^m + B^m)$ is a multiple of $(A + B)$.

   (b) Let $k > 1$ be a positive integer. Prove that if $2^k + 1$ is a prime number, then k must be a power of 2. Hint: proof by contradiction.

3. (a) Prove (by induction or some other clever technique) that for r a positive integer, we have
   $$1 + 2 + 4 + ... + 2^r = 2^{r+1} - 1.$$

   (b) Let k be a positive integer. Let $p = 2^k - 1$. Let $n = p \times 2^{k-1}$. Prove that if p is prime, then n is a perfect number. Hint: Since the prime factorization of n is exactly $p \times 2^{k-1}$, just find all the positive integer factors of n and then add up all the proper factors (those less than n).