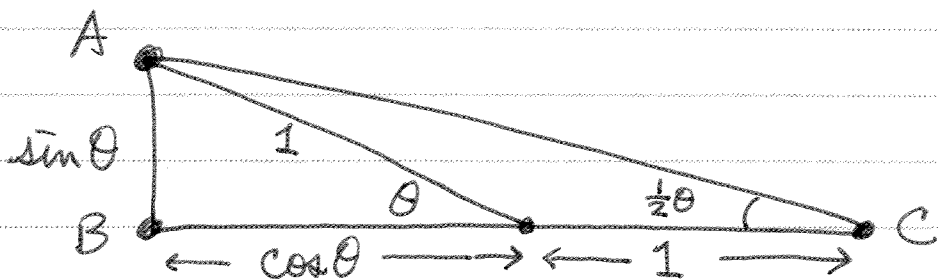


Homework Chapter 4

#1



Use the above diagram to derive a formula for
(a) $\cos \frac{1}{2}\theta$ and simplify it to $\sqrt{\frac{1+\cos\theta}{2}}$

(b) $\sin \frac{1}{2}\theta$ and simplify it to $\sqrt{\frac{1-\cos\theta}{2}}$

(c) $\tan \frac{1}{2}\theta$

#2 By considering the inscribed regular n -gon and circumscribed regular n -gon, prove that

$$n \sin \frac{\pi}{n} < \pi < n \tan \frac{\pi}{n}.$$

#3 $\cos 30^\circ = \frac{\sqrt{3}}{2} \Rightarrow \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \Rightarrow \cos \frac{\pi}{12} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

Then #2 with $n=24$ yields

$$24 \sin \frac{\pi}{24} < \pi < 24 \tan \frac{\pi}{24}$$

yields
by #1, $24 \sqrt{\frac{1 - \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}}{2}} < \pi < 24 \frac{\sqrt{1 - \frac{\sqrt{3}}{2}}}{1 + \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}}$

which yields $3.13263 < \pi < 3.15966$.

Do the same with $n=48$.