Math 499 – D. Yuen

Homework: Chapter 8

1. Review of how to sum a telescopic series rigorously: By definition,
   $\sum_{k=1}^{\infty}\frac{2}{k^2+k}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{k^2+k}$. We have
   $\sum_{k=1}^{n}\frac{2}{k^2+k}=\sum_{k=1}^{n}(\frac{2}{k}-\frac{2}{k+1})=\frac{2}{2}-\frac{2}{3}+\frac{2}{3}-\frac{2}{4}\ldots+\frac{2}{n}-\frac{2}{n+1}=\frac{2}{2}-\frac{2}{n+1}$.
   Thus $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{k^2+k}=\lim_{n\to\infty}\sum_{k=1}^{n}(\frac{2}{k}-\frac{2}{k+1})=\lim_{n\to\infty}(\frac{2}{2}-\frac{2}{n+1})=1$.
   Use this technique to find the value of the sum $\sum_{k=2}^{\infty}\frac{2}{k^2-1}$ by using partial fractions and then writing out the series to see that everything cancels except a few terms.

2. Find the value of the sum $\sum_{k=1}^{\infty}\frac{8}{4k^2-1}$ again by using partial fractions and canceling as in #1.

3. Using the inequality proven in the talk that $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2^k}\geq\frac{k+2}{2}$ for $k\geq 0$, find a value of n such that guarantees $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\geq 1000$.

4. Using the inequality that $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\geq\int_1^{n+1}\frac{1}{x}dx$ for $n\geq 1$, find a value of n such that guarantees $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\geq 1000$.