NAME:______________________________

Math 230 Midterm #1
February 7, 2014

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

• You may NOT use a calculator.

• Show all of your work.

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1. True or False (Just answer true or false, you don’t need to explain your answer).

(a) [2 points] \( T \subseteq A \) if and only if \( T \in 2^A \).

(b) [2 points] There is no \( x \) such that \( x \subseteq \{x\} \).

(c) [2 points] If \( x \) is a real number and \( x^2 < 0 \), then \( x \) is a perfect number.

(d) [2 points] Two right triangles that have hypotenuses of the same length have the same area.

(e) [2 points] \( \exists x, \forall y, xy = 0 \).

(f) [2 points] \( \forall x, \exists y, xy = 0 \).

(g) [2 points] \( \mathbb{N} \in 2^\mathbb{Z} \).

(h) [2 points] \( \{2\} \subseteq \{\{1\}, \{2\}, \{3\}\} \).

(i) [2 points] If \( A \) and \( B \) are sets then \( 2^A \subseteq 2^B \).

(j) [2 points] A negation of the statement “There is a natural number that is prime and even” can be phrased as “All natural numbers that are prime are odd”.
2. For the following pairs of statements $A$, $B$, write $a$ if the statement “If $A$, then $B$” is true, write $b$ if the statement “If $B$, then $A$” is true, write $c$ if the statement ”$A$ if and only if $B$ is true”, and write $d$ if none of the statements are true. You should write all that apply. Note that in the following, $x$ and $y$ are integers.

(a) [5 points] $A$: $xy = 0$. $B$: $x = 0$ and $y = 0$.

(b) [5 points] $A$: Lines $l_1$ and $l_2$ are parallel. $B$: Lines $l_1$ and $l_2$ are perpendicular.

(c) [5 points] $A$: Joe is a grandfather. $B$: Joe is male.

(d) [5 points] $A$: $x < 0$ $B$: $x^3 < 0$. 
3. Proofs:

(a) [10 points] Let $x$ be an integer. Prove that $x$ is odd if and only if there is an integer $b$ such that $x = 2b - 1$. 
(b) [5 points] For real numbers $a$ and $b$, prove that if $0 < a < b$, then $a^2 < b^2$.

(c) [5 points] Let $A, B$ and $C$ be sets satisfying $A \subseteq B$ and $B \subseteq C$. Prove that $A \subseteq C$. 
4. Find counterexamples to disprove the following statements:

(a) [5 points] If $a$, $b$ and $c$ are positive integers with $a | (bc)$, then $a | b$ or $a | c$.

(b) [5 points] Two right triangles have the same area if and only if the lengths of their hypotenuses are the same.

(c) [5 points] For real numbers $a$ and $b$, if $a < b$, then $a^2 < b^2$.

(d) [5 points] Let $A$ and $B$ be sets. Then $(A \cup B) - B = A$. 
5. Boolean Algebra

(a) [5 points] Prove or disprove the following Boolean expression identity:

\[(x \land y) \lor (x \land \neg y) = x.\]

(b) [5 points] Besides the classic Boolean operations \(\land, \lor, \neg, \rightarrow, \leftarrow\), we have others, an example of one is the “nand” operation denoted by \(\bar{\land}\). We define \(x\bar{\land}y\) to be \(\neg(x \land y)\). Construct a truth table for \(\bar{\land}\).
(c) [5 points] Prove or disprove that \( \bar{\wedge} \) is commutative.

(d) [5 points] Prove or disprove that \( \bar{\wedge} \) is associative.
6. In my comic book library I have 15 Daredevil paperbacks, 12 Spider-man paperbacks and 3 Batman paperbacks

(a) [5 points] In how many different ways can these trade paperbacks be arranged on a bookshelf?

(b) [5 points] In how many different ways can these trade paperbacks be arranged on a bookshelf if all the books of the same character are grouped together?
7. Write out the following sets by listing their elements between curly braces.

(a) [5 points] \( \{ x \in \mathbb{N} : x \leq 10 \text{ and } 3|x \} \).

(b) [5 points] \( \{ x \in \mathbb{Z} : x^2 = 4 \} \).

(c) [5 points] \( \{ x \in \mathbb{Z} : 10|x \text{ and } x|100 \} \).

(d) [5 points] \( \{ x : x \subseteq \{1, 2, 3, 4, 5\} \text{ and } |x| \leq 1 \} \).
8. Let $A \times B = \{(1, 2), (1, 3), (1, 7), (2, 2), (2, 3), (2, 7), (6, 2), (6, 3), (6, 7)\}$.

(a) [5 points] What is $A \cup B$?

(b) [5 points] What is $A \cap B$?

(c) [5 points] What is $A - B$?

(d) [5 points] What is $A \Delta B$?