Homework 2a Solutions
Math 230

10.12: Let $C = \{x \in \mathbb{Z} : x|12\}$ and let $D = \{x \in \mathbb{Z} : x|36\}$. Prove that $C \subseteq D$.

Proof. Let $c \in C$. Then $c|12$, so there exists an integer k such that $12 = kc$. Since $12 = kc$, then $36 = 3kc = (3k)c = mc$. Since $k \in \mathbb{Z}$, then $m = 3k \in \mathbb{Z}$, so 36 is a multiple of c, so $c \in D$. Therefore $C \subseteq D$.

$\square$

10.13: Generalize the previous problem. Let c and d be integers and let $C = \{x \in \mathbb{Z} : x|c\}$ and $D = \{x \in \mathbb{Z} : x|d\}$.

Find and prove necessary and sufficient conditions for $C \subseteq D$.

Proof. Suppose $C \subseteq D$. Note that $c \in C$ because $c|c$. Since $C \subseteq D$, then $c \in D$. Therefore $c|d$. So a necessary condition for $C \subseteq D$ is that $c|d$.

Now suppose $c|d$. Let $x \in C$. Since $x \in C$, then $x|c$, so $c = kx$ for some integer k. We also know that $c|d$, so $d = mc$ for some integer m. Since $d = mc$ and $c = kx$, then $d = mkx = (mk)x$. Since mk is an integer, then $x|d$. Therefore $x \in D$. So $c|d$ implies $C \subseteq D$, so $c|d$ is a sufficient condition for $C \subseteq D$.

In summary $C \subseteq D$ if and only if $c|d$.

$\square$