Practice Exam Math 230

1. True or False (Just answer true or false, you don't need to explain your answer):

   (a) Every positive integer is either prime or composite.

   (b) Every integer is positive or negative.

   (c) If x is an integer and x is prime and $x > 2$ then x is odd.

   (d) Let x and y be integers such that $x^2 = y^2$, then $x = y$.

   (e) $\{2\} \in 2^{\mathbb{Z}}$.

   (f) Let A be a set, then $\emptyset \subseteq A$.

   (g) $2 \in \{\{1\}, \{2\}\}$.

   (h) If you pick a guinea pig up by its tail, then its eyes will pop out.

   (i) If p and q are prime then $p + q$ is composite.

   (j) There exists an even number n such that $n + 1$ is also even.

2. The following statement is false: "If x, y and z are integers and $x > y$, then $xz > yz$". Do the following:

   (a) Find a counterexample.

   (b) Modify the statement by adding an extra condition on z that will make the conclusion true.

3. Prove or disprove that the Boolean expressions $x \to \neg y$ and $\neg(x \to y)$ are logically equivalent.

4. Prove that the sum of three consecutive positive integers is a multiple of 3.

5. Let a be an integer. Prove that if $a \geq 3$, then $a^2 > 2a + 1$.

6. The call sign for a radio station in the United States is a list of three or four letters, such as WJHU or WJZ. The first letter must be a W or a K, and there is no restriction on the other letters. In how many ways can the call sign of the radio station be formed?

7. In how many ways can we arrange a standard deck of 52 cards so that all cards in a given suit appear contiguously.

8. Prove or disprove: $2^{A \cap B} = 2^A \cap 2^B$.

9. True or false (Just answer true or false, you don't need to explain your answer):

   (a) $\forall x \in \mathbb{Z}, x^2 \geq x$.

   (b) $\exists x \in \mathbb{Z}, x^3 = x$.

   (c) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leq x$.

   (d) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x \leq y$.

10. Suppose A and B are finite sets. Given that $|A| = 10, |A \cup B| = 15$, and $|A \cap B| = 3$, determine $|B|$.

11. Let A, B, C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.