# Practice Exam 1
## Math 230
## SOLUTIONS

(1) a) False, 1 is neither
b) False, 0 is neither
c) True, all primes $>2$ are odd.
d) False, $x=-y$ also satisfies the equation.
e) True, $2^{\mathbb{Z}}$ is the set of subsets of $\mathbb{Z}$ and $\{2\}\subseteq\mathbb{Z}$ so $\{2\}\in 2^{\mathbb{Z}}$.
f) True, $\emptyset$ is a subset of every set.
g) False, 2 is not an element of $\{\{1\},\{2\}\}$.
h) True, because guinea pigs don't have tails.
i) False, $2+3=5$ and 5 is not composite.
j) False, If n is even, n+1 is odd.

(2) a) $z=-1$ is the counterexample.
$-x<-z$ because $z<x$ so $(-1)(x)$ is not $>(-1)(z)$.

b) "If $x, y, z$ are integers and $x>y$ and $z>0$, then $xz>yz$."

(I added $z>0$).

(3)

| $x$ | $y$ | $\neg y$ | $x\to y$ | $x\to\neg y$ | $\neg(x\to y)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | T | F | T | T |
| F | T | F | T | T | F |
| F | F | T | T | T | F |

The columns are different, so they are not equivalent.

Alternative proof:
They are not the same.
Indeed if $x=F$ and $y=F$ then $x\to\neg y$ is true
while $\neg(x\to y)$ is false.

(4) "If $n$ is a positive integer then $n+(n+1)+(n+2)$ is a multiple of 3".

Let $n$ be a positive integer.

$$n+(n+1)+(n+2) = 3n+3$$
$$= 3(n+1)$$
$$= 3c$$

where $c$ is the integer $(n+1)$

Therefore $3 \mid n+(n+1)+(n+2)$.

Therefore $n+(n+1)+(n+2)$ is a multiple of 3. $\boxtimes$

(5) Let $a$ be an integer such that $a \geq 3$.

Then $a > 0$ and $a-2 \geq 0$.
Therefore $a(a-2) > 0$
so $a^2 - 2a > 1$
so $a^2 \geq 2a+1$.
Therefore $a^2 > 2a+1$ $\boxtimes$

(6) 2 choices for first letter, 26 choices for second, 26 choices for third (any letter) and 27 for the fourth one since it can be empty or any letter of the alphabet.

Therefore $\boxed{2 \times 26 \times 26 \times 27}$

(7) 4! ways of ordering the suits, then each suit can be permuted 13! ways so

$$\boxed{4! \times 13! \times 13! \times 13! \times 13!}$$

(8) It is true, let's prove it.

$(\Rightarrow)$ Let $x \in 2^{A \cap B}$ then $x \subseteq A \cap B$.

So $x$ is a set, let's call it $C$.

$C \subseteq A \cap B$, Therefore $C \subseteq A$ and $C \subseteq B$,

Therefore $C \in 2^A \cap 2^B$, therefore $x \in 2^A \cap 2^B$.

($\Leftarrow$)

Let $x \in 2^A \cap 2^B$

then $x$ is a subset of both $A$ and $B$,

therefore $x$ is a subset of $A \cap B$.

Therefore $x \in 2^{A\cap B}$ $\boxtimes$

(9) a) True. Indeed if $x<0$ then $x^2 \geq 0$ so $x^2 > x$.

If $x > 1$ then $x-1 > 0$ so $x(x-1) > 0$

so $x^2 > x$.

The only cases left (for integers) are

$x=0$ and $x=1$ in both cases we have $x^2 \geq x$.

($0^2 = 0$ and $1^2 = 1$)

b) True. $x=1$ satisfies $1^3 = 1$, i.e. $x^3 = x$.

c) True. $x \leq x$ always no matter what $x$ is or $y$

d) True. Let $x=1$. If $y \in \mathbb{N}$

Then $y \in \{1, 2, 3, \ldots\}$

so $y \geq 1$ so $y \geq x$.

So the $x$ exists.

(10) $|A \cup B| = |A| + |B| - |A \cap B|$

$15 = 10 + |B| - 3$

so

$\boxed{|B| = 8}$

(11) ($\Rightarrow$)

Let $x \in A \times (B \cap C)$

then $x = (a,b)$ where $a \in A$ and $b \in B \cap C$.

Since $b \in B \cap C$ then $b \in B$ and $b \in C$

so ($a \in A$) and ($b \in B$ and $b \in C$)

implies ($a \in A$ and $b \in B$) and ($a \in A$ and $b \in C$).

Therefore $(a,b) \in A \times B$ and $(a,b) \in A \times C$

so $x \in (A \times B) \cap (A \times C)$.

$(\Leftarrow)$ Let $x = (a,b) \in (A \times B) \cap (A \times C)$.

Since $(a,b) \in A \times B \Rightarrow a \in A$ and $b \in B$

Since $(a,b) \in A \times C \Rightarrow a \in A$ and $b \in C$

Therefore $(a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

so $(a \in A)$ and $(b \in B$ and $b \in C)$

so $(a,b) \in A \times (B \cap C)$

so $x \in A \times B \cap C$

Alternative Proof: Let $x = (a,b)$.

$(a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

translates to $\cancel{(m \wedge b) \wedge (a \wedge c)}$

$(m \wedge n) \wedge (m \wedge p)$ where $m = a \in A$, $n = b \in B$, $p = b \in C$

But

$(m \wedge n) \wedge (m \wedge p) = m \wedge (n \wedge p)$

So

$(a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C) = (a \in A)$ and $(b \in B$ and $b \in C)$

so

$$(A \times B) \cap (A \times C) = A \times (B \cap C)$$