

Practice Exam 2

- Prove that the following identities are true for all positive integers n :
 - $1 + 5 + 9 + \dots + (4n - 3) = 2n^2 - n$.
 - $1 + 10 + 10^2 + \dots + 10^n = \frac{10^{(n+1)} - 1}{9}$.
- Prove that the following inequalities are true:
 - $e^n > n + 7$, for $n \geq 3$.
 - $n^2 \geq 6n + 2$, for $n \geq 7$.
- Prove by induction that the sum of the angles of a convex n -gon (with $n \geq 3$) is $180(n - 2)$ degrees.
- For each of the following relations defined on the set $\{1, 2, 3\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.
 - $R = \{(1, 1), (2, 2), (3, 3)\}$.
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 - $R = \{(1, 2), (1, 3), (2, 3), (2, 2)\}$.
- For each equivalence relation below, find the requested equivalence class.
 - $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find $[1]$.
 - $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$. Find $[4]$.
 - R is has-the-same-tens-digit-as on the set $\{x \in \mathbb{Z} : 100 < x < 200\}$. Find $[123]$.
 - R is has-the-same-size-as on $2^{\{1, 2, 3, 4, 5\}}$. Find $[\{1, 3\}]$.
 - $R = \{(a, b) : a, b \in \mathbb{Z}, 1 \leq a \leq 30, 1 \leq b \leq 30, 7|(b - a)\}$ on $\{1, 2, 3, \dots, 30\}$. Find $[3]$.
- Let R be the “is similar” relation on triangles, i.e. if A and B are triangles, then $(A, B) \in R$ if and only if the angles of triangle A are the same as the angles of triangle B . Prove that R is an equivalence relation.
- A poker hand consists of 5 cards chosen from a standard deck of 52 cards.
 - How many different poker hands are there?
 - How many poker hands are three-of-a-kind (no full houses)?
 - How many poker hands are a straight (could be a straight flush)?
 - How many poker hands are just 1 pair and nothing else?
 - How many poker hands contain two pairs but are not a full house?
- How many rectangles can be formed from an $m \times n$ chess board? For example, for a 2×2 chess board, there are nine possible rectangles.

REMARK: More questions to study include the homework exercises, more exercises from the sections covered and I specially recommend practicing proofs involving relations. Make sure you know how to prove something is an equivalence relation, how to prove things using induction and how to count things.