NAME: ______________________________

MATH 230 MIDTERM #2

October 28, 2013

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	20	
3	10	
4	10	
5	10	
6	20	
7	15	
Total:	105	

1. True or False (Just answer true or false, you don't need to explain your answer):
   (a) [2 points] $23 \equiv 3 \bmod 10$.

   (b) [2 points] $111 \equiv 95 \bmod 7$.

   (c) [2 points] The is-less-than relation is transitive.

   (d) [2 points] If a relation is symmetric then it is not antisymmetric.

   (e) [2 points] A non-empty relation cannot be both reflexive and irreflexive.

   (f) [2 points] If a relation is reflexive and symmetric it must also be transitive.

   (g) [2 points] A relation R on a set A is antisymmetric if and only if $R \cap R^{-1} \subseteq \{(a, a) : a \in A\}$

   (h) [2 points] Let A and B be two statements. To prove "If A, then B" by contradiction one assumes A is true and B is false, then one proves that this is impossible.

   (i) [2 points] To prove by induction that for all $n \geq 3$, $e^n > n + 7$, one must show that the inequality is true when $n = 3$.

   (j) [2 points] The difference between a proof by strong induction and a proof by induction is that the base cases are dealt with differently.

2. Prove that the following are true:

   (a) [5 points] $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$, for all positive integers n.

   (b) [5 points] $3+7+11+\ldots+(4n-1) = 2n^2+n$, for all positive integers n.

(c) [5 points] $n^2 > 2n + 1$, for all integers $n \geq 3$.

(d) [5 points] $2^n > n^2$, for all integers $n \geq 5$.

3. For each of the following statements, write the first sentences of a proof by contradiction (you should not attempt to complete the proofs).

   (a) [5 points] Prove that the sum of any four consecutive integers is not divisible by 4.

   (b) [5 points] Let P be a finite set of (three or more) points in the plane and suppose that any three points in P are colinear. (Three points are colinear if all three lie in the same line. For example, the points $(0,0),(1,1),(7,7)$ are colinear, but $(0,0),(1,1),(3,4)$ are not.) Prove that all points in P lie in a common line.

4. Prove the following statements:

   (a) [5 points] Prove that the sum of any four consecutive integers is not divisible by 4.

   (b) [5 points] Let P be a finite set of (three or more) points in the plane and suppose that any three points in P are colinear. (Three points are colinear if all three lie in the same line. For example, the points $(0,0),(1,1),(7,7)$ are colinear, but $(0,0),(1,1),(3,4)$ are not.) Prove that all points in P lie in a common line.

5. For each of the following relations defined on the set $\{1, 2, 3, 4\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.

   (a) [5 points]

$$R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(4,4),(1,4),(4,1)\}$$

   (b) [5 points]

$$R = \{(1,1),(2,2),(3,3),(1,4),(4,4),(1,3),(4,3)\}$$

6. For each equivalence relation below, find the requested equivalence classes.

   (a) [5 points] R is the relation

   $$R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(4,4),(1,4),(4,1),(2,4),(4,2)\}$$

   on the set $\{1,2,3,4\}$. Find $[4]$ and $[1]$.

   (b) [5 points] R is has-the-same-size-as relation on $2^{\{1,2,3,4,5\}}$. Find $[\{1,2,3\}]$.

(c) [5 points] $R = \{(x, y) : x, y \in \mathbb{Z} \text{ and } |x| = |y|\}$. Find $[-3]$ and $[0]$.

(d) [5 points] R is the relation on the set of points on the plane where the point (a, b) is related to the point (c, d) if and only if $a^2 + b^2 = c^2 + d^2$ (for all a, b, c, d real numbers). What figure in the plane does $[(0, 1)]$ represent?

7. [15 points] Let R be the relation on the set of points on the plane where the point (a, b) is related to the point (c, d) if and only if $a^2 + b^2 = c^2 + d^2$ (for all a, b, c, d real numbers). Prove that R is an equivalence relation on the set of points on the plane.