Name: ____________________________

Math 230 Midterm #2

March 3, 2014

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	20	
3	10	
4	20	
5	15	
6	15	
7	10	
Total:	110	

1. True or False (Just answer true or false, you don't need to explain your answer):

   (a) [2 points] Let R be an equivalence relation on the set A. Let $a \in A$, then $a \in [a]$.

   (b) [2 points] Let R be an equivalence relation on A. If $a \in A$ and $b \in A$, then the intersection of $[a]$ and $[b]$ is empty.

   (c) [2 points] Let R be a symmetric relation. Then $R = R^{-1}$.

   (d) [2 points] If a relation is symmetric then it is not antisymmetric.

   (e) [2 points] There is a relation that is reflexive and irreflexive.

   (f) [2 points] If a relation is reflexive and symmetric it must also be transitive.

   (g) [2 points] A relation R on a set A is antisymmetric if and only if $R \cap R^{-1} \subseteq \{(a, a) : a \in A\}$

   (h) [2 points] Let R be an equivalence relation on a set A. The equivalence classes of R form a partition of the set A.

   (i) [2 points] The difference between a proof by strong induction and a proof by induction is that the base cases are dealt with differently.

   (j) [2 points] The is-less-than-or-equal-to relation is an equivalence relation.

2. Prove that the following are true:

(a) [5 points] $1 + 3 + 3^2 + \ldots + 3^n = \frac{3^{n+1}-1}{2}$, for all positive integers n.

(b) [5 points] $1+5+9+\ldots+(4n-3) = 2n^2 - n$, for all positive integers n.

(c) [5 points] $n^2 > 2n + 1$, for all integers $n \geq 3$.

(d) [5 points] $2^n > n^2$, for all integers $n \geq 5$.

3. For each of the following relations defined on the set $\{1, 2, 3, 4\}$ determine whether they are reflexive, irreflexive, symmetric, antisymmetric and/or transitive.

   (a) [5 points]

$$R = \{(1, 1), (2, 2), (3, 3), (1, 4), (4, 4), (1, 3), (4, 3)\}.$$

   (b) [5 points]

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (4, 4), (1, 4), (4, 1)\}.$$

4. For each equivalence relation below, find the requested equivalence classes.

   (a) [5 points] $R = \{(x, y) : x, y \in \mathbb{Z} \text{ and } |x| = |y|\}$. Find $[-3]$ and $[0]$.

   (b) [5 points] R is has-the-same-size-as relation on $2^{\{1,2,3,4,5\}}$. Find $[\{1, 2, 3\}]$.

(c) [5 points] R is the relation

$$R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(4,4),(1,4),(4,1),(2,4),(4,2)\}$$

on the set $\{1,2,3,4\}$. Find [1] and [2].

(d) [5 points] R is the relation on the set of points on the plane where the point (a,b) is related to the point (c,d) if and only if $a^2+b^2=c^2+d^2$ (for all a,b,c,d real numbers). What figure in the plane does $[(0,1)]$ represent?

5. [15 points] Let R be the relation on the set of points on the plane where the point (a, b) is related to the point (c, d) if and only if $a^2 + b^2 = c^2 + d^2$ (for all a, b, c, d real numbers). Prove that R is an equivalence relation on the set of points on the plane.

6. Number of relations

   (a) [5 points] Let $A = \{1\}$. How many different relations on A are there?

   (b) [5 points] Let $A = \{1, 2\}$. How many different relations on A are there?

   (c) [5 points] Let $A = \{1, 2, 3 \ldots, n\}$. How many different relations on A are there?

7. A special type of door lock has a panel with five buttons labeled with the digits 1 through 5. This lock is opened by a sequence of three actions. Each action consists of either pressing one of the buttons or pressing a pair of them simultaneously. For example, 12-4-3 is a possible combination. The combination 12-4-3 is the same as 21-4-3 because both the 12 and the 21 simply mean to press buttons 1 and 2 simultaneously.

   (a) [5 points] How many combinations are possible?

   (b) [5 points] How many combinations are possible if no digit is repeated in the combination?