Poker Hands

July 24, 2013

A deck of cards has 52 cards, 13 numbers ($\mathbf{A}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}, \mathbf{8}, \mathbf{9}, \mathbf{10}, \mathbf{J}, \mathbf{Q}, \mathbf{K}$), 4 suits (*spades,clubs,hearts,diamonds*). A poker hand consists of 5 cards from the deck. A pair means two cards have the same number. Three of a kind, means three of the same number, similarly defined for four of a kind. A Full House is when the poker hand consists of a three-of-a-kind and a pair (example **AA222**). A straight means you have five consecutive numbers, the first possible straight is $\mathbf{A}-\mathbf{2}-\mathbf{3}-\mathbf{4}-\mathbf{5}$, the last one is $\mathbf{10}-\mathbf{J}-\mathbf{Q}-\mathbf{K}-\mathbf{A}$. A flush means the five cards are of the same suit. A straight flush means the hand is both a flush and a straight.

1. How many poker hands are there? (For example, one poker hand is **A** of hearts, **2** of spades, **J** of hearts, **5** of clubs and **A** of diamonds.

$$\binom{52}{5} = 2598960$$

2. How many poker hands are there with just one pair?

$$\frac{52\times 3}{2} \times \frac{48\times 44\times 40}{3!} = 1098240$$

or

$$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$$

pick the pair

pick the suits for the pair

pick the other three numbers

pick their suits.

3. How many poker hands with two pairs are there?

$$\frac{\frac{52\times 3}{2} \times \frac{48\times 3}{2}}{2} \times 44 = 123552$$

or

$$\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}.$$

4. How many poker hands with three of a kind are there (and not a full house)?

$$\frac{52 \times 3 \times 2}{3!} \times \frac{48 \times 44}{2!} = 54\,912$$

or

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2$$

5. How many poker hands with four of a kind are there?

$$\frac{52 \times 3 \times 2 \times 1}{4!} \times 48 = 624$$

or

$$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{}$$

6. How many poker hands are a full house?

$$\frac{52 \times 3 \times 2}{3!} \times \frac{48 \times 3}{2!} = 3744$$

or

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$$

7. How many poker hands are a straight flush?

$$10(4) = 40$$

$$10 \quad \left\{ \begin{array}{l} A2345 \\ 23456 \\ \vdots \\ TJQKA \end{array} \right.$$

4 suits for each

8. How many poker hands are a straight but not a flush?

$$10\left(4^5\right) - 40 = 10200.$$

The straights

Setting the suits for the straights

9. How many poker hands are a flush but not a straight?

$$4\binom{13}{5} - 40 = \frac{52\cdot 12\cdot 11\cdot 10\cdot 9}{5!} - 40 = 5108$$

10. How many poker hands are left (no pair, no flush, no straight)?

$\binom{52}{5}$ − pairs − two pair − three of a kind − straigh − flush − straight flush

$= 1\ 302\ 540.$

11. Can you give a hierarchy to the poker hands from less common to most common?

	#	%
Straight Flush	40	0.00154 %
Four of a kind	624	0.024 %
Full House	3744	0.144 %
Flush	5108	0.1965 %
Straight	10200	0.392465 %
Three of a kind	54912	2.113 %
Two Pair	123552	4.754 %
Pair	1098240	42.257 %
Nothing	1302540	50.12 %