

Polymath REU: A Program to Encourage Undergraduate Math Research Across the Globe

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LAKE FOREST
COLLEGE

- Started in 2009 by Timothy Gowers.
- Idea: Use online collaboration to advance math problems that are “modular”.
- More than 15 Polymath projects have been done. Polymath8 (2013) is probably the most famous.

Polymath8 Summary

- Y. Zhang proved there are infinitely many primes p such that the next prime q satisfies $q - p \leq 70000000$.
- Polymath8 attacked the problem and brought down the number to $q - p \leq 4680$ after a few months of work.
- Maynard-Tao had an alternative proof that showed $q - p \leq 600$.
- Polymath8b brought down this number to $q - p \leq 246$ (current record).

Polymath8 Progress

Date	ϖ or (ϖ, δ)	k_0	H
Aug 10 2005		6 [EH]	16 [EH] ([Goldston-Pintz-Yildirim ↗])
May 14 2013	1/1,168 (Zhang ↗)	3,500,000 (Zhang ↗)	70,000,000 (Zhang ↗)
May 21			63,374,611 (Lewko ↗)
May 28			59,874,594 (Trudgian ↗)
May 30			59,470,640 (Morrison ↗) 58,885,998? (Tao ↗) 59,093,364 (Morrison ↗) 57,554,086 (Morrison ↗)
May 31		2,947,442 (Morrison ↗) 2,618,607 (Morrison ↗)	48,112,378 (Morrison ↗) 42,543,038 (Morrison ↗) 42,342,946 (Morrison ↗)
Jun 1			42,342,924 (Tao ↗)
Jun 2		866,605 (Morrison ↗)	13,008,612 (Morrison ↗)
Jun 3	1/1,040? (v08ltu ↗)	341,640 (Morrison ↗)	4,982,086 (Morrison ↗) 4,802,222 (Morrison ↗)

Polymath8 Progress

Jun 3	1/1,040? (v08ltu ↗)	341,640 (Morrison ↗)	4,982,086 (Morrison ↗) 4,802,222 (Morrison ↗)
Jun 4	1/224?? (v08ltu ↗) 1/240?? (v08ltu ↗)		4,801,744 (Sutherland ↗) 4,788,240 (Sutherland ↗)
Jun 5		34,429? (Paldi ↗ /v08ltu ↗) 34,429? (Tao ↗ /v08ltu ↗ /Harcos ↗)	4,725,021 (Elsholtz ↗) 4,717,560 (Sutherland ↗) 397,110? (Sutherland ↗) 4,656,298 (Sutherland ↗) 389,922 (Sutherland ↗) 388,310 (Sutherland ↗) 388,284 (Castrыck ↗) 388,248 (Sutherland ↗) 388,188 ↗ (Sutherland ↗) 387,982 (Castrыck ↗) 387,974 (Castrыck ↗)
Jun 6	1/488,3/9272 (Pintz ↗) 1/552 (Pintz ↗ , Tao ↗)	60,000 [±] (Pintz ↗) 52,295 [±] (Peake ↗) 11,123 (Tao ↗)	387,960 (Angelveit ↗) 387,910 ↗ (Sutherland ↗) 387,904 (Angelveit ↗) 387,814 ↗ (Sutherland ↗) 387,766 ↗ (Sutherland ↗) 387,754 ↗ (Sutherland ↗) 387,620 ↗ (Sutherland ↗)

Polymath8 Progress

			768,534 [±] (Pintz ↗)
			113,520 ↗ ? (Angeltveit ↗)
			109,314 ↗ ? (Angeltveit/Sutherland ↗)
			707,328 [±] ↗ (Sutherland ↗)
			108,990 ↗ (Sutherland ↗)
			113,462 [±] ↗ (Sutherland ↗)
			112,302 [±] ↗ (Sutherland ↗)
			112,272 [±] ↗ (Sutherland ↗)
			116,386 [±] (Sun ↗)
			108,978 ↗ (Sutherland ↗)
			108,634 ↗ (Sutherland ↗)
			108,632 ↗ (Castruck ↗)
			108,600 ↗ (Sutherland ↗)
			108,570 ↗ (Castruck ↗)
		11,018 (Tao ↗)	108,556 ↗ (Sutherland ↗)
	(1/538, 1/660) (v08ltu ↗)	10,721 (v08ltu ↗)	108,550 ↗ (xfxie ↗)
	(1/538, 31/20444) (v08ltu ↗)	10,719 (v08ltu ↗)	275,424 ↗ (Sutherland ↗)
Jun 7	(1/942, 19/27004) (v08ltu ↗)	25,111 (v08ltu ↗)	108,540 ↗ (Sutherland ↗)
	$828\omega + 172\delta < 1$ (v08ltu ↗ /Green ↗)	26,024? (vo8ltu ↗)	275,418 ↗ (Sutherland ↗)
			275,404 ↗ (Sutherland ↗)
			275,292 ↗ (Castruck- Sutherland ↗)

Polymath8 Progress

Jun 24	$(134 + \frac{2}{3})\varpi + 28\delta \leq 1?$ (v08ltu ↗) $140\varpi + 32\delta < 1?$ (Tao ↗) $1/88??$ (Tao ↗) $1/74??$ (Tao ↗)	1,268? (v08ltu ↗)	10,206? ↗ (Engelsma ↗)
Jun 25	$116\varpi + 30\delta < 1?$ (Fouvry-Kowalski-Michel-Nelson ↗ /Tao ↗)	1,346? (Hannes ↗) 502?? (Trevine ↗) 1,007? (Hannes ↗)	10,876 ↗ ? (Engelsma ↗) 3,612 ↗ ?? (Engelsma ↗) 7,860 ↗ ? (Engelsma ↗)
Jun 26	$116\varpi + 25.5\delta < 1?$ (Nielsen ↗) $(112 + \frac{4}{7})\varpi + (27 + \frac{6}{7})\delta < 1?$ (Tao ↗)	962? (Hannes ↗)	7,470 ↗ ? (Engelsma ↗)
Jun 27	$108\varpi + 30\delta < 1?$ (Tao ↗)	902? (Hannes ↗)	6,966 ↗ ? (Engelsma ↗)
Jul 1	$(93 + \frac{1}{3})\varpi + (26 + \frac{2}{3})\delta < 1?$ (Tao ↗)	873? (Hannes ↗) 872? (xfxie ↗)	6,712? ↗ (Sutherland ↗) 6,696? ↗ (Engelsma ↗)
Jul 5	$(93 + \frac{1}{3})\varpi + (26 + \frac{2}{3})\delta < 1$ (Tao ↗)	720 (xfxie ↗ /Harcos ↗)	5,414 ↗ (Engelsma ↗)
Jul 10	7/600? (Tao ↗)		
Jul 19	$(85 + \frac{5}{7})\varpi + (25 + \frac{5}{7})\delta < 1?$		

- REU stands for Research Experience for Undergraduates.
- The National Science Foundation (NSF) sponsors many REU programs across the country.
- Since funding is from the NSF, international students don't have many REU options.
- Due to COVID, many REUs were canceled in 2020.

- A group of mathematicians from across the country created Polymath REU in 2020: Kira Adaricheva, Ben Brubaker, Pat Devlin, Steven J. Miller, Vic Reiner, Alexandra Seceleanu, Adam Sheffer, Yunus Zeytuncu
- The idea is to have professors mentor students from across the globe. To be accepted into the program, students needed to show they had taken one upper level class and have one letter recommendation. Most students accepted (in 2020 it was 300 out of 350).
- Each professor gets between 15 and 25 students to mentor on a research project. Students can contribute as much as they can, or as little as they want, but the opportunity is presented to them.

My experience in 2021

- I was one of the research mentors in 2021.
- I recruited a student of mine, Maximilano Sánchez Garza, to be my research assistant.
- 8 week program.
- I led a probability research project with the following students:
 - Martín Prado Guerra (Colombia)
 - Lisa Lin (Rice)
 - Eli Sun
 - Qizhao Rong (Cuny)
 - Sammi Matoush (Washington University in St. Louis)
 - Rachael Ren (Washington)
 - Nasser Malibari (UNSW Sydney, Australia)
 - Chandramauli Chakraborty (IIT, India)
 - Isaiah Milbank (Minnesota)
 - Qizhou Fang (UC Irvine)
 - Julian Burden (Gettysburg)

Our project

- Suppose you have an urn with R red balls and W white balls.
- In probability there are two classic scenarios to sample n balls from the urn.
 - When you take out a ball, you record what you got and you return the ball in the urn. (Called “sampling with replacement”)
 - When you take out a ball, you record what you got and you keep it. (Called “sampling without replacement”)
- We studied what happens when you choose to not replace the white balls, but replace the red balls. This was studied in a recent paper and we were able to answer their open questions.
- We also studied a generalization, where you keep a white ball with probability w and keep a red ball with probability r . For this version, we found a recursive formula to find expected value quickly and we proved some bounds.

The International Math Olympiad

- Started in 1959 with 7 countries: Romania, Hungary, Czechoslovakia, Bulgaria, Poland, USSR, East Germany.
- USA first competes in 1974.
- Mexico technically first competes in 1981, but really starts in 1987.
- 107 countries competed in 2021. The largest IMO had 112 countries in 2019.
- The exam consists of two days of exams. Each day has 3 problems and students have 4.5 hours to solve them.

My role

- Competed in the Mexican Math Olympiad in 2000 and 2001.
- Was a coach for the Chihuahua team most years between 2003 and 2015.
- Started coaching the Mexican team in 2016. I am currently one of the coaches.
- Have been team leader or deputy leader for Mexico at several international competitions.
- I was the editor in chief of the Asian Pacific Math Olympiad in 2018-2019.
- I am currently a co-editor in chief-elect of the United States Math Olympiad. My role goes from 2022 to 2025.

Thank you!