What's so RATIONAL about the ALPHABET?

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Walking on Real Numbers

- **2013 study by Aragón Artacho, Bailey, Borwein, & Borwein**
- **Used random walks and other graphical methods to examine the normality of mathematical constants, such as e, π, Champernowne's number etc.**
    - **Also examined the walks of bounded rational numbers**

1049012271677499437486619280565448601
1756735849156087616684838084314435844
2528755516292470277595555704537156793
3058783247729772021770818187965906373
5767487981422801328592027861019258140
5713574870471229026746515131280595419
399750420206138037382233895971339195
/
1612226962694290912940490066273549214
2988075572546851235339571846519135301
3488143140175045399694454793530120643
3327267097007933052629203035092097360
4509554561365966493250783914647728401
2385651374295294530896122681527488756
565807616241078807518459942193877483

Base Conversion

- **People typically think of numbers in Base 10**
  {0, 1, 2, 3, 4,
  5, 6, 7, 8, 9}
- **For the purpose of our project, we chose to use Base 4**
  {0, 1, 2, 3}
- **Why?**
  We can code them as the cardinal directions (N, S, E, W)
  0 = east / right
  1 = north / up
  2 = west / left
  3 = south / down

Example in Base 4:
419636198 (base 10)

- Start with base 10 number and divide by the desired base (in this case, base 4)
- **Note the remainder and divide the new number by the desired base again**
- Continue this step and note the remainders until you can divide no further
  (x/y = 0 R x)
- Read the remainders **in reverse order** for the base conversion

121000302033212
(base 4)

4 / 419636198	104909049	R 2
4 / 104909049	26227262	R 1
4 / 26227262	6556815	R 2
4 / 6556815	1639203	R 3
4 / 1639203	409800	R 3
4 / 409800	102450	R 0
4 / 102450	25612	R 2
4 / 25612	6403	R 0
4 / 6403	1600	R 3
4 / 1600	400	R 0
4 / 400	100	R 0
4 / 100	25	R 0
4 / 25	6	R 1
4 / 6	1	R 2
4 / 1	0	R 1

Random Walks

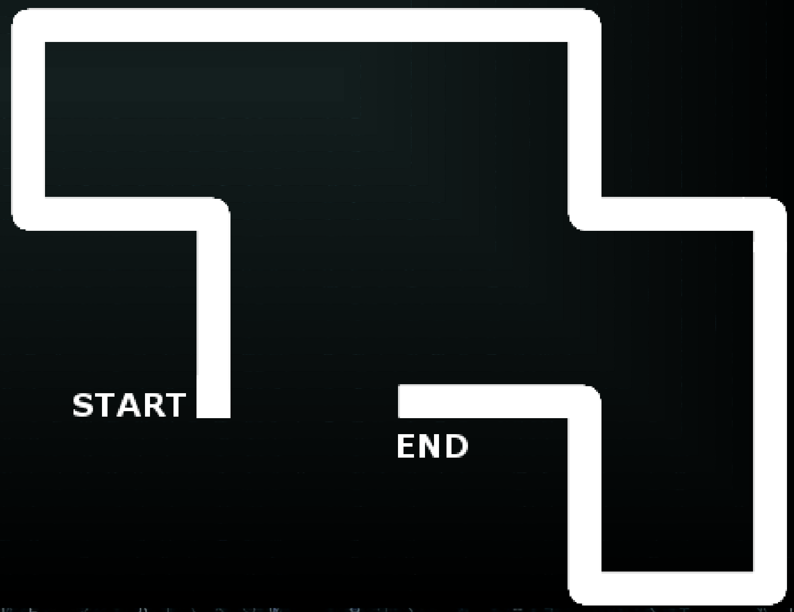
- **Depending on the base used in the random walk, each number is assigned a direction that the computer is told to move one unit in when it appears in a rational number (which could be "randomly" generated or a specific input).**

Example in Base 4:
0 = move 1 unit 0 radians (right)
1 = move 1 unit π/2 radians (up)
2 = move 1 unit π radians (left)
3 = move 1 unit 3π/2 radians (down)

419636198 (base 10)
121000302033212 (base 4)

Translates to:
↑,←,↑,→,→,→,↓,→,←,→,↓,↓,←,↑,←

The result:



The Rational Alphabet

OUR GOAL: to find rational numbers that represent each letter of the alphabet.

- **Rather than looking first for the numbers that gave us the desired random walk, we worked backwards from base 4 and converted the desired walks into base 10 quotients.**

{2,0,0,0,1,2,3,1,2,3,1,2,3,1,1,0,3,1,0,3,1,0,3
,1,1,2,3,1,2,3,1,2,3,1,1,0,3,1,0,3,1,0,3,1,1,2,
3,1,2,3,1,2,3,1,1,0,3,1,0,3,1,0,3,1,0,1,2,1,0,
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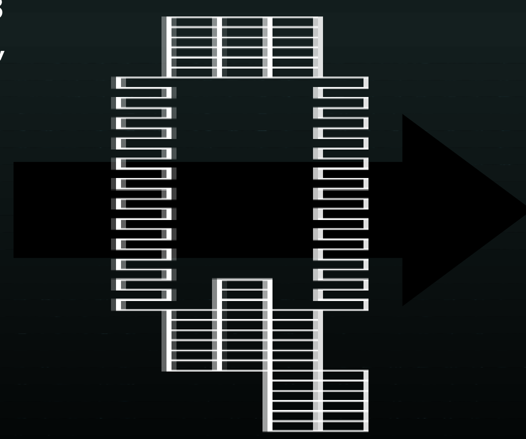
3,0,3,2,3,0,3,2,3,0,3,2,3,0,3,2,3,0,0,1,0,3,1,
1,2,3,1,1,0,3,3,3,3,3,3,3,3,3,3,0,0,1,2,3,1,2,
3,3,0,1,3,0,1,3,3,2,1,3,2,1,3,3,0,1,3,0,1,3,3,
2,1,3,2,1,3,3,0,1,3,0,1,3,2,2,1,1,1,1,1,1,2,2}



≈ **0.501674**

260213063964275792487388206834868455003776874127286321853429097935333249232391674474608322354406723630818470312210699072326139758849409885013777046876597167306844106307946137070253
/
518689446110124119814050982961395143876555779030304612499457166211331601426613518299963381118387974286024735826412598647799393884426471913485859354264245460882647725425188690460672

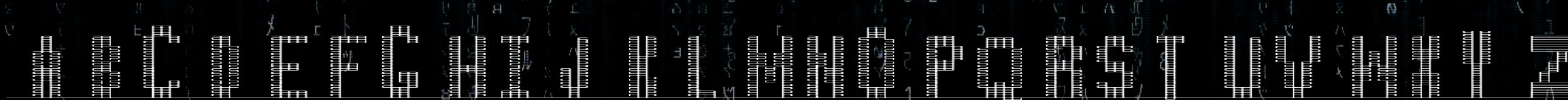
The Rational Alphabet

- **We then wrote a code that would allow us to combine the letters to print words and phrases**

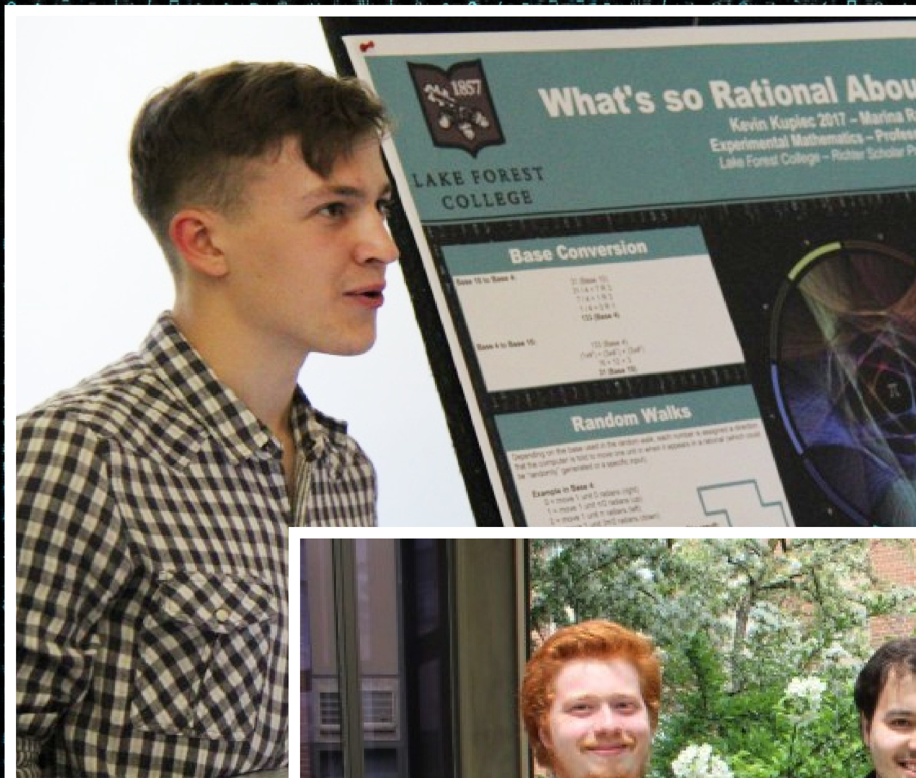
```
c[" "] = {rspace, nspace};

randomWalk[x_, b_, n_] := Module[{list = {{0, 0}},
   point = {0, 0},
   digits = RealDigits[x, b, n][[1]]},
  Do[
   point[[1]] =
    point[[1]] + N[Cos[(2 Pi / b) * digits[[i]]]];
   point[[2]] =
    point[[2]] + N[Sin[(2 Pi / b) * digits[[i]]]];
   AppendTo[list, point]
   , {i, 1, Length[digits]}
  ];
  ListLinePlot[list, Axes → False,
   AspectRatio → Automatic]
 ]
```

```
WriteWord[word_, m_] :=
 Module[{x, n, b = 4, w = 8,
   letters = Characters[word]},
  x =
   (Sum[c[letters[[i]]][[1]] /
       (4 ^ (Sum[c[letters[[j]]][[2]] + w,
           {j, 1, i - 1}])), {i, 1, Length[letters]}]
     (2 /
        4 ^ (Sum[c[letters[[j]]][[2]] + w,
            {j, 1, Length[letters]}] - w))
      (1 - 1 / 4 ^ ((Length[letters] - 1) w)) / 3);
  n = Sum[c[letters[[i]]][[2]],
     {i, 1, Length[letters]}] +
    2 (Length[letters] - 1) w;
  x = x * 4 ^ n / (4 ^ n - 1);
  randomWalk[x, b, n]
   Print[{x, n}]]
```



QUESTIONS?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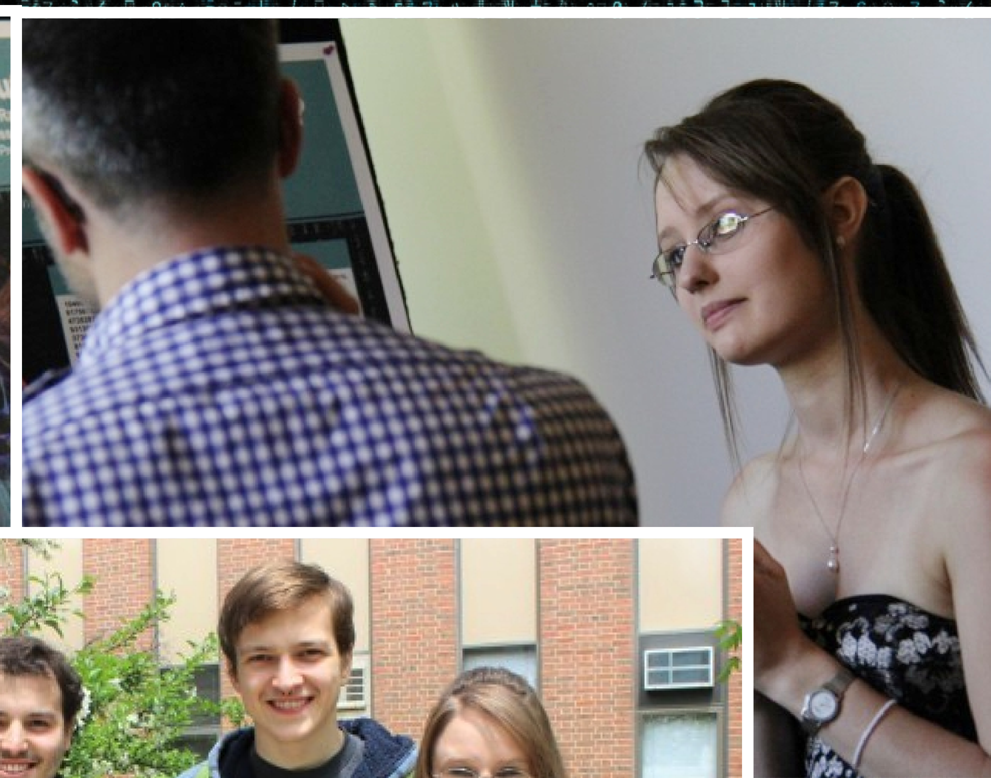

References

Aragón Artacho, F. J., Bailey, D. H., Borwein, J. M., & Borwein, P. B. (2013). Walking on real numbers.