Review 1 of Chapter 10

March 16, 2012

1. Find the Taylor polynomials of degree 4 approximating the following functions for x near 0.

   (a) $\sqrt{1+x}$,

   (b) $\arctan x$,

   (c) $\frac{1}{\sqrt{1+x}}$.

2. Find the Taylor polynomial of degree 4 approximating the following functions for x near 1.

   (a) $\sqrt{1-x}$,

   (b) $\sqrt{1+x}$,

   (c) $\ln(x^2)$.

3. The function $f(x)$ is approximated near $x = 0$ by the third degree polynomial

$$P_3(x) = 2 - x - \frac{x^2}{3} + 2x^3.$$

   Give the values of $f(0), f'(0), f''(0)$ and $f'''(0)$.

4. Show how you can use the Taylor approximation $\sin(x) \approx x - \frac{x^3}{3!}$, for x near 0, to explain why

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

5. Find the first four terms of the Taylor series for the following functions about $x = a$.

   (a) $\sin x$, for $a = \frac{\pi}{4}$.

   (b) $\frac{1}{x}$ for $a = 2$.

6. Find an expression for the general term of the series:

   (a) $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots.$

   (b) $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots.$

   (c) $e^{x^2} = 1 + x^2 + \dfrac{x^4}{2!} + \dfrac{x^6}{3!} + \dfrac{x^8}{4!} + \ldots.$

7. Calculate the following sums b recognizing the Taylor series evaluaion:

   (a) $1 - \dfrac{1}{3!} + \dfrac{1}{5!} - \dfrac{1}{7!} + \ldots.$

   (b) $1 - \dfrac{100}{2!} + \dfrac{10000}{4!} - \dfrac{1000000}{6!} + \ldots.$

   (c) $1 - 0.1 + (0.1)^2 - (0.1)^3 + \ldots.$

8. Find the radius of convergence of the Taylor series for $\sqrt{1+x}$ around $x = 0$.