Review 2 of Chapter 10

March 30, 2012

1. Find the first 4 nonzero terms of the Taylor series about 0 for the following functions.
   (a) $\sqrt{1+3x}$,
   (b) $\cos(\theta^2)$,
   (c) $t\sin(t^2) - t^3$.
2. Expand $\dfrac{1}{x+2}$ in terms of $\dfrac{x}{2}$.
3. For values of x near 0, order the following three functions from smallest to largest:
   (a) $1 + \sin x$,
   (b) e^x,
   (c) $\dfrac{1}{\sqrt{1-2x}}$.
4. Use Taylor series to explain the patterns in the digits in the following expansions:
   (a) $\dfrac{1}{0.98} = 1.02040816\ldots$,
   (b) $\left(\dfrac{1}{0.99}\right)^2 = 1.020304050607\ldots$.
5. Approximate the following numbers using the third degree Taylor polynomial about 0 and then find an estimate on how big the error can be:
   (a) e^1, using $f(x) = e^x$.
   (b) $\ln(2)$, using $f(x) = \ln(1+x)$.
   (c) $\sqrt{2}$, using $f(x) = \sqrt{1+x}$.

6. Let $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. Let $P_n(x)$ be the n-th degree Taylor polynomial of $f(x)$ about 0. Given x between -1 and 1, find how big n must be to guarantee that the error $|f(x) - P_n(x)|$ is smaller than:

   (a) 1.5

   (b) 0.5.

   (c) 0.1.

7. True/False:

   (a) If $f(x)$ and $g(x)$ have the same Taylor polynomial of degree 2 near $x = 0$, then $f(x) = g(x)$.

   (b) The Taylor series for $x^3 \cos x$ about $x = 0$ has only odd powers.

   (c) The Taylor series for $f(x)$ converges everywhere f is defined.

   (d) A Taylor polynoial for f near $x = a$ touches the graph of f only at $x = a$.

   (e) If $f^{(n)}(0) \geq n!$ for all n, then the Taylor series for f near $x = 0$ diverges at $x = 0$.