Review of Chapter 7

February 16, 2012

1. Find the following integrals:
   (a) $\int e^t + 5e^{5t}\,dt.$
   (b) $\int x^2 \cos 3x\,dx.$
   (c) $\int_1^{64} x^{3/2} + x^{2/3}\,dx.$
   (d) $\int \frac{t+1}{t^2}\,dt.$
   (e) $\int_1^3 x(x^2+1)^{70}\,dx.$
   (f) $\int \frac{\sin \ln x}{x}\,dx.$
   (g) $\int \frac{dr}{r^2 - 100}.$
   (h) $\int \frac{x^3}{x^2+3x+2}\,dx.$
   (i) $\int \frac{(2x-1)e^{x^2}}{e^x}\,dx.$
   (j) $\int \frac{3x+1}{x(x^2-1)}\,dx.$
2. A function is defined by $f(t) = t^2$ for $0 \le t \le 1$ and $f(t) = 2 - t$ for $1 < t \le 2$. Compute $\int_0^2 f(t)\,dt$.
3. What, if anything, is wrong with the following calculation?

$$\int_{-2}^{2} \frac{1}{x^2}\,dx = \frac{1}{x}\,\Big|_{-2}^{2} = -\frac{1}{2} - \left(-\frac{1}{-2}\right) = -1.$$

4. Find $LEFT(6), RIGHT(6), MID(6)$ and $TRAP(6)$ to approximate

$$\int_0^6 e^{-x^2}\,dx.$$

5. Answer True or False to the following statements.

(a) $\int f'(x)\cos(f(x))\,dx = \sin(f(x)) + C.$

(b) $\int \frac{dx}{f(x)} = \ln|f(x)| + C.$

(c) The trapezoid rule approximation is never exact.

(d) The midpoint rule approximation to $\int_0^1 (y^2-1)\,dy$ is always smaller than the exact value of the integral.

(e) If $f(x)$ is continuous and positive for $x > 0$ and if $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

(f) If $\int_0^\infty f(x)\,dx$ and $\int_0^\infty g(x)\,dx$ both converge, then $\int_0^\infty f(x)+g(x)\,dx$ converges.

(g) If $\int_0^\infty f(x)\,dx$ converges, then $\int_0^\infty 7f(x)\,dx$ converges.

(h) If $\int_0^\infty f(x)\,dx$ converges, then $\int_0^\infty f(7x)\,dx$ converges.

(i) Doubling n decreases the difference $LEFT(n) - RIGHT(n)$ by exactly the factor $\frac{1}{2}$.

(j) If $0 < f' < g'$ everywhere, then the error in aproximating $\int_a^b f(x)\,dx$ by $LEFT(n)$ is less than the error in approximating $\int_a^b g(x)\,dx$ by $LEFT(n)$.