5.3.8 $A + B$ will not necessarily be orthogonal, because the columns may not be unit vectors. For example, if $A = B = I_n$, then $A + B = 2I_n$, which is not orthogonal.

5.3.28 Write $L(\vec{x}) = A\vec{x}$; by Definition 5.3.1, A is an orthogonal $n \times n$ matrix, so that $A^T A = I_n$, by Theorem 5.3.7. Now $L(\vec{v}) \cdot L(\vec{w}) = (A\vec{v}) \cdot (A\vec{w}) = (A\vec{v})^T A\vec{w} = \vec{v}^T A^T A\vec{w} = \vec{v}^T I_n \vec{w} = \vec{v}^T \vec{w} = \vec{v} \cdot \vec{w}$, as claimed. Note that we have used Theorems 5.3.6 and 5.3.9a.

5.3.29 We will use the fact that L preserves length (by Definition 5.3.1) and the dot product (by Exercise 28):

$$\angle(L(\vec{v}), L(\vec{w})) = \arccos \frac{L(\vec{v}) \cdot L(\vec{w})}{\|L(\vec{v})\| \|L(\vec{w})\|} = \arccos \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} = \angle(\vec{v}, \vec{w}).$$

5.3.30 If $L(\vec{x}) = \vec{0}$, then $\|L(\vec{x})\| = \|\vec{x}\| = 0$, so that $\vec{x} = \vec{0}$. Therefore, $\ker(L) = \{\vec{0}\}$.

By Theorem 3.3.7, $\dim(\text{im}(L)) = m - \dim(\ker(L)) = m$.

Since $\mathbb{R}^n$ has an m-dimensional subspace (namely, $\text{im}(L)$), the inequality $m \leq n$ holds.

The transformation L preserves right angles (the proof of Theorem 5.3.2 applies), so that the columns of A are orthonormal (since they are $L(\vec{e}_1), \ldots, L(\vec{e}_m)$).

Therefore, we have $A^T A = I_m$ (the proof of Theorem 5.3.7 applies).

Since the vectors $\vec{v}_1, \ldots, \vec{v}_m$ form an orthonormal basis of $\text{im}(A)$, the matrix AA^T represents the orthogonal projection onto $\text{im}(A)$, by Theorem 5.3.10.

A simple example of such a transformation is $L(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x}$, that is, $L \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$.

5.3.31 Yes! If A is orthogonal, then so is A^T, by Exercise 11. Since the columns of A^T are orthogonal, so are the rows of A.

5.3.32 a No! As a counterexample, consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ (see Exercise 30).

b Yes! More generally, if A and B are $n \times n$ matrices such that $BA = I_n$, then $AB = I_n$, by Theorem 2.4.8c.

5.3.40 An orthonormal basis of W is $\vec{u}_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -0.1 \\ 0.7 \\ -0.7 \\ 0.1 \end{bmatrix}$ (see Exercise 5.2.9).

By Theorem 5.3.10, the matrix of the projection onto W is QQ^T, where $Q = [\,\vec{u}_1 \quad \vec{u}_2\,]$.

$$QQ^T = \frac{1}{100}\begin{bmatrix} 26 & 18 & 32 & 24 \\ 18 & 74 & -24 & 32 \\ 32 & -24 & 74 & 18 \\ 24 & 32 & 18 & 26 \end{bmatrix}$$

5.3.56 Yes and yes (see Exercise 57).

5.3.57 Yes, L is linear, since $L(A+B) = (A+B)^T = A^T + B^T = L(A) + L(B)$ and $L(kA) = (kA)^T = kA^T = kL(A)$.
Yes, L is an isomorphism; the inverse is the transformation $R(A) = A^T$ from $\mathbb{R}^{n\times m}$ to $\mathbb{R}^{m\times n}$.

5.4.20 Using Theorem 5.4.6, we find $\vec{x}^* = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\vec{b} - A\vec{x}^* = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

Note that $\vec{b} - A\vec{x}^*$ is perpendicular to the two columns of A.

5.4.21 Using Theorem 5.4.6, we find $\vec{x}^* = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{b} - A\vec{x}^* = \begin{bmatrix} -12 \\ 36 \\ -18 \end{bmatrix}$, so that $\|\vec{b} - A\vec{x}^*\| = 42$.

5.4.22 Using Theorem 5.4.6, we find $\vec{x}^* = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\vec{b} - A\vec{x}^* = \vec{0}$. This system is in fact consistent and $\vec{x}^*$ is the exact solution; the error $\|\vec{b} - A\vec{x}^*\|$ is 0.

5.4.30 We attempt to solve the system

$$\begin{matrix} c_0 + 0c_1 = 0 \\ c_0 + 0c_1 = 1 \\ c_0 + 1c_1 = 1 \end{matrix}, \text{or} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

This system cannot be solved exactly; the least-squares solution is $\begin{bmatrix} c_0^* \\ c_1^* \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$. The line that fits the data points best is $f^*(t) = \frac{1}{2} + \frac{1}{2}t$.

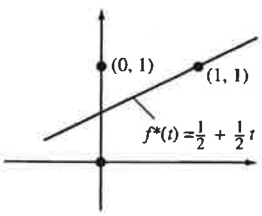

Figure 5.18: for Problem 5.4.30.

The line goes through the point (1, 1) and "splits the difference" between (0, 0) and (0, 1). See Figure 5.18.

5.4.31 We want $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$ such that

$$\begin{matrix} 3 = c_0 + 0c_1 \\ 3 = c_0 + 1c_1 \\ 6 = c_0 + 1c_1 \end{matrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}.$$

Since $\ker \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \{\vec{0}\}, \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}^* = \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{3}{2} \end{bmatrix}$ so $f^*(t) = 3 + \frac{3}{2}t$. (See Figure 5.19.)

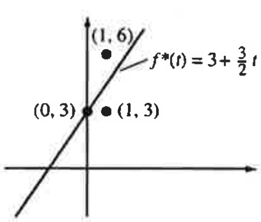

Figure 5.19: for Problem 5.4.31.

5.4.32 We want $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$ of $f(t) = c_0 + c_1 t + c_2 t^2$ such that

$$\begin{aligned} 27 &= c_0 + 0c_1 + 0c_2 \\ 0 &= c_0 + 1c_1 + 1c_2 \\ 0 &= c_0 + 2c_1 + 4c_2 \\ 0 &= c_0 + 3c_1 + 9c_2 \end{aligned} \text{ or } \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 27 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If we call the coefficient matrix A, we notice that $\ker(A) = \{\vec{0}\}$ so

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}^* = (A^TA)^{-1}A^T \begin{bmatrix} 27 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25.65 \\ -28.35 \\ 6.75 \end{bmatrix} \text{ so } f^*(t) = 25.65 - 28.35t + 6.75t^2.$$

5.4.**33** We want $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$ such that

$$\begin{aligned} 0 &= c_0 + \sin(0)c_1 + \cos(0)c_2 \\ 1 &= c_0 + \sin(1)c_1 + \cos(1)c_2 \\ 2 &= c_0 + \sin(2)c_1 + \cos(2)c_2 \\ 3 &= c_0 + \sin(3)c_1 + \cos(3)c_2 \end{aligned} \text{ or } \begin{bmatrix} 1 & 0 & 1 \\ 1 & \sin(1) & \cos(1) \\ 1 & \sin(2) & \cos(2) \\ 1 & \sin(3) & \cos(3) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}.$$

Since the coefficient matrix has kernel $\{\vec{0}\}$, we compute $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}^*$ using Theorem 5.4.6, obtaining

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}^* \approx \begin{bmatrix} 1.5 \\ 0.1 \\ -1.41 \end{bmatrix} \text{ so } f^*(t) \approx 1.5 + 0.1 \sin t - 1.41 \cos t.$$

5.4.**34** We want $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$ such that

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & \sin(0.5) & \cos(0.5) & \sin(1) & \cos(1) \\ 1 & \sin(1) & \cos(1) & \sin(2) & \cos(2) \\ 1 & \sin(1.5) & \cos(1.5) & \sin(3) & \cos(3) \\ 1 & \sin(2) & \cos(2) & \sin(4) & \cos(4) \\ 1 & \sin(2.5) & \cos(2.5) & \sin(5) & \cos(5) \\ 1 & \sin(3) & \cos(3) & \sin(6) & \cos(6) \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \end{bmatrix}$$

Since the columns of the coefficient matrix are linearly independent, its kernel is $\{\vec{0}\}$. We can use Theorem 5.4.6 to compute $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \approx \begin{bmatrix} 1.5 \\ 0.109 \\ -1.537 \\ 0.303 \\ 0.043 \end{bmatrix}$ so $f^*(t) \approx 1.5 + 0.109 \sin(t) - 1.537 \cos(t) + 0.303 \sin(2t) + 0.043 \cos(2t)$.

5.4.**35** a The ijth entry of $A_n^T A_n$ is the dot product of the ith row of A_n^T and the jth column of A_n, i.e.

$$A_n^T A_n = \begin{bmatrix} n & \sum_{i=1}^n \sin a_i & \sum_{i=1}^n \cos a_i \\ \sum_{i=1}^n \sin a_i & \sum_{i=1}^n \sin^2 a_i & \sum_{i=1}^n \sin a_i \cos a_i \\ \sum_{i=1}^n \cos a_i & \sum_{i=1}^n \sin a_i \cos a_i & \sum_{i=1}^n \cos^2 a_i \end{bmatrix} \text{ and } A_n^T \vec{b} = \begin{bmatrix} \sum_{i=1}^n g(a_i) \\ \sum_{i=1}^n g(a_i) \sin a_i \\ \sum_{i=1}^n g(a_i) \cos a_i \end{bmatrix}.$$

b $$\lim_{n\to\infty} \tfrac{2\pi}{n} A_n^T A_n = \begin{bmatrix} 2\pi & \int_0^{2\pi} \sin t\,dt & \int_0^{2\pi} \cos t\,dt \\ \int_0^{2\pi} \sin t\,dt & \int_0^{2\pi} \sin^2 t\,dt & \int_0^{2\pi} \sin t \cos t\,dt \\ \int_0^{2\pi} \cos t\,dt & \int_0^{2\pi} \sin t \cos t\,dt & \int_0^{2\pi} \cos^2 t\,dt \end{bmatrix} = \begin{bmatrix} 2\pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{bmatrix}$$

and $$\lim_{n\to\infty} \tfrac{2\pi}{n} A_n^T \vec{b} = \begin{bmatrix} \int_0^{2\pi} g(t)\,dt \\ \int_0^{2\pi} g(t) \sin t\,dt \\ \int_0^{2\pi} g(t) \cos t\,dt \end{bmatrix}$$

(Here $\frac{2\pi}{n} = \Delta t$ so $\lim_{n\to\infty} \frac{2\pi}{n} \sum_{i=1}^n \cos(t_i) = \lim_{n\to\infty} \sum_{i=1}^n \cos(t_i)\Delta t = \int_0^{2\pi} \cos t\,dt$ for instance. All other limits are obtained similarly.)

c $$\begin{bmatrix} c \\ p \\ q \end{bmatrix} = \lim_{n\to\infty} \begin{bmatrix} c_n \\ p_n \\ q_n \end{bmatrix} = \begin{bmatrix} 2\pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{bmatrix}^{-1} \begin{bmatrix} \int_0^{2\pi} g(t)\,dt \\ \int_0^{2\pi} g(t) \sin t\,dt \\ \int_0^{2\pi} g(t) \cos t\,dt \end{bmatrix} = \begin{bmatrix} \frac{1}{2\pi} \int_0^{2\pi} g(t)\,dt \\ \frac{1}{\pi} \int_0^{2\pi} g(t) \sin t\,dt \\ \frac{1}{\pi} \int_0^{2\pi} g(t) \cos t\,dt \end{bmatrix}$$ and $f(t) = c + p \sin t + q \cos t$, where c, p, q are given above.

5.4.**36** We want $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ such that

$$\begin{aligned} a + b \sin\left(\frac{2\pi}{366} 32\right) + c \cos\left(\frac{2\pi}{366} 32\right) &= 10 \\ a + b \sin\left(\frac{2\pi}{366} 77\right) + c \cos\left(\frac{2\pi}{366} 77\right) &= 12 \\ a + b \sin\left(\frac{2\pi}{366} 121\right) + c \cos\left(\frac{2\pi}{366} 121\right) &= 14 \end{aligned}$$

$$a + b\sin\left(\frac{2\pi}{366}152\right) + c\cos\left(\frac{2\pi}{366}152\right) \quad = \quad 15$$

Using $A = \begin{bmatrix} 1 & \sin\left(\frac{2\pi}{366}32\right) & \cos\left(\frac{2\pi}{366}32\right) \\ 1 & \sin\left(\frac{2\pi}{366}77\right) & \cos\left(\frac{2\pi}{366}77\right) \\ 1 & \sin\left(\frac{2\pi}{366}121\right) & \cos\left(\frac{2\pi}{366}121\right) \\ 1 & \sin\left(\frac{2\pi}{366}152\right) & \cos\left(\frac{2\pi}{366}152\right) \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 10 \\ 12 \\ 14 \\ 15 \end{bmatrix}$, we compute $\begin{bmatrix} a \\ b \\ c \end{bmatrix}^*$

$= (A^TA)^{-1}A^T\vec{b} \approx \begin{bmatrix} 12.26 \\ 0.431 \\ -2.899 \end{bmatrix}$ and $f^*(t) \approx 12.26 + 0.431\sin\left(\frac{2\pi}{366}t\right) - 2.899\cos\left(\frac{2\pi}{366}t\right)$.

5.4.**37** a We want c_0, c_1 such that

$$\begin{aligned} c_0 + c_1(35) &= \log 35 \\ c_0 + c_1(46) &= \log 46 \\ c_0 + c_1(59) &= \log 77 \\ c_0 + c_1(69) &= \log 133 \end{aligned} \quad \text{or} \quad \underbrace{\begin{bmatrix} 1 & 35 \\ 1 & 46 \\ 1 & 59 \\ 1 & 69 \end{bmatrix}}_{A} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \log 35 \\ \log 46 \\ \log 77 \\ \log 133 \end{bmatrix}}_{\vec{b}}$$

so $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}^* = (A^TA)^{-1}A^T\vec{b} \approx \begin{bmatrix} 0.915 \\ 0.017 \end{bmatrix}$ so $\log(d) \approx 0.915 + 0.017t$.

b $d \approx 10^{0.915} \cdot 10^{0.017t} \approx 8.22 \cdot 10^{0.017t}$

c If $t = 88$ then $d \approx 258$. Since the Airbus has only 93 displays, new technologies must have rendered the old trends obsolete.

5.4.**38** We want $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$ such that

$$\begin{aligned} 110 &= c_0 + 2c_1 + c_2 \\ 180 &= c_0 + 12c_1 + 0c_2 \\ 120 &= c_0 + 5c_1 + c_2 \\ 160 &= c_0 + 11c_1 + c_2 \\ 160 &= c_0 + 6c_1 + 0c_2 \end{aligned} \quad \text{or} \quad \begin{bmatrix} 1 & 2 & 1 \\ 1 & 12 & 0 \\ 1 & 5 & 1 \\ 1 & 11 & 1 \\ 1 & 6 & 0 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 110 \\ 180 \\ 120 \\ 160 \\ 160 \end{bmatrix}.$$

The least-squares solution is $\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}^* = \begin{bmatrix} 125 \\ 5 \\ -25 \end{bmatrix}$, so that $w^* = 125 + 5h - 25g$.

For a general population, we expect c_0 and c_1 to be positive, since c_0 gives the weight of a 5′ male, and increased height should contribute positively to the weight. We expect c_2 to be negative, since females tend to be lighter than males of equal height.

5.4.**39** a We want $\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$ such that

6.1.4 Fails to be invertible; since $\det\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix} = 8 - 8 = 0$.

6.1.18 $\det\begin{bmatrix} 0 & 1 & k \\ 3 & 2k & 5 \\ 9 & 7 & 5 \end{bmatrix} = 30 + 21k - 18k^2 = -3(k-2)(6k+5)$. So k cannot be 2 or $-\frac{5}{6}$.

6.1.26 $\det(A - \lambda I_2) = \det \begin{bmatrix} 4-\lambda & 2 \\ 2 & 7-\lambda \end{bmatrix} = (4-\lambda)(7-\lambda) - 4 = (\lambda - 8)(\lambda - 3) = 0$ if λ is 3 or 8.

6.1.27 $A - \lambda I_3$ is a lower triangular matrix with the diagonal entries $(2-\lambda)$, $(3-\lambda)$ and $(4-\lambda)$. Now, $\det(A - \lambda I_3) = (2-\lambda)(3-\lambda)(4-\lambda) = 0$ if λ is 2, 3 or 4.

6.1.28 $A - \lambda I_3$ is an upper triangular matrix with the diagonal entries $(2-\lambda)$, $(3-\lambda)$ and $(5-\lambda)$. Now, $\det(A - \lambda I_3) = (2-\lambda)(3-\lambda)(5-\lambda) = 0$ if λ is 2, 3 or 5.

Chapter 7

Section 7.1

7.1.1 If $\vec{v}$ is an eigenvector of A, then $A\vec{v} = \lambda\vec{v}$.

Hence $A^3\vec{v} = A^2(A\vec{v}) = A^2(\lambda\vec{v}) = A(A\lambda\vec{v}) = A(\lambda A\vec{v}) = A(\lambda^2\vec{v}) = \lambda^2 A\vec{v} = \lambda^3\vec{v}$, so $\vec{v}$ is an eigenvector of A^3 with eigenvalue λ^3.

7.1.2 We know $A\vec{v} = \lambda\vec{v}$ so $\vec{v} = A^{-1}A\vec{v} = A^{-1}\lambda\vec{v} = \lambda A^{-1}\vec{v}$, so $\vec{v} = \lambda A^{-1}\vec{v}$ or $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$.

Hence $\vec{v}$ is an eigenvector of A^{-1} with eigenvalue $\frac{1}{\lambda}$.

7.1.3 We know $A\vec{v} = \lambda\vec{v}$, so $(A + 2I_n)\vec{v} = A\vec{v} + 2I_n\vec{v} = \lambda\vec{v} + 2\vec{v} = (\lambda + 2)\vec{v}$, hence $\vec{v}$ is an eigenvector of $(A + 2I_n)$ with eigenvalue $\lambda + 2$.

7.1.4 We know $A\vec{v} = \lambda\vec{v}$, so $7A\vec{v} = 7\lambda\vec{v}$, hence $\vec{v}$ is an eigenvector of $7A$ with eigenvalue 7λ.

7.1.5 Assume $A\vec{v} = \lambda\vec{v}$ and $B\vec{v} = \beta\vec{v}$ for some eigenvalues λ, β. Then $(A + B)\vec{v} = A\vec{v} + B\vec{v} = \lambda\vec{v} + \beta\vec{v} = (\lambda + \beta)\vec{v}$ so $\vec{v}$ is an eigenvector of $A + B$ with eigenvalue $\lambda + \beta$.

7.1.6 Yes. If $A\vec{v} = \lambda\vec{v}$ and $B\vec{v} = \mu\vec{v}$, then $AB\vec{v} = A(\mu\vec{v}) = \mu(A\vec{v}) = \mu\lambda\vec{v}$

7.1.7 We know $A\vec{v} = \lambda\vec{v}$ so $(A - \lambda I_n)\vec{v} = A\vec{v} - \lambda I_n\vec{v} = \lambda\vec{v} - \lambda\vec{v} = \vec{0}$ so a nonzero vector $\vec{v}$ is in the kernel of $(A - \lambda I_n)$ so $\ker(A - \lambda I_n) \neq \{\vec{0}\}$ and $A - \lambda I_n$ is not invertible.

7.1.8 We want all $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ hence $\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, i.e. the desired matrices must have the form $\begin{bmatrix} 5 & b \\ 0 & d \end{bmatrix}$.

7.1.9 We want $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for any λ. Hence $\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \end{bmatrix}$, i.e., the desired matrices must have the form $\begin{bmatrix} \lambda & b \\ 0 & d \end{bmatrix}$, they must be upper triangular.

7.1.10 We want $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, i.e. the desired matrices must have the form $\begin{bmatrix} 5 - 2b & b \\ 10 - 2d & d \end{bmatrix}$.

7.1.11 We want $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$. So, $2a + 3b = -2$ and $2c + 3d = -3$. Thus, $b = \frac{-2-2a}{3}$, and $d = \frac{-3-2c}{3}$. So all matrices of the form $\begin{bmatrix} a & \frac{-2-2a}{3} \\ c & \frac{-3-2c}{3} \end{bmatrix}$ will fit.

7.1.12 Solving $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 2\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ we get $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} t \\ -\frac{3}{2}t \end{bmatrix}$ (with $t \neq 0$) and

solving $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 4\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ we get $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix}$ (with $t \neq 0$).

Section 7.2

7.2.1 $\lambda_1 = 1$, $\lambda_2 = 3$ by Theorem 7.2.2.

7.2.2 $\lambda_1 = 2$ (Algebraic multiplicity 2)

$\lambda_2 = 1$ (Algebraic multiplicity 2), by Theorem 7.2.2.

7.2.3 $\det(A - \lambda I_2) = \det \begin{bmatrix} 5-\lambda & -4 \\ 2 & -1-\lambda \end{bmatrix} = \lambda^2 - 4\lambda + 3 = (\lambda - 1)(\lambda - 3) = 0$ so $\lambda_1 = 1, \lambda_2 = 3$.

7.2.10 $f_A(\lambda) = (1+\lambda)^2(1-\lambda)$ so $\lambda_1 = -1$ (Algebraic multiplicity 2), $\lambda_2 = 1$.