

Worksheet #1, Proofs by Induction, Math 28

September 5, 2012

1. Prove that for every positive integer n the following is true:

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. For $r \neq 1$, prove

$$1 + r + r^2 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1},$$

for all positive integers n .

3. Calculate

$$1 \times 1000 + 2 \times 999 + 3 \times 998 + \dots + 999 \times 2 + 1000 \times 1.$$

4. Prove that the number of subsets of a set with n elements is 2^n . (Note: The empty set and the set itself count as subsets).