Mathematical Olympiads

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Mathematical Olympiads

- Annual Mathematics competition for High School Students

- First IMO held in Bucharest, Romania in 1959 (insert cheer from Iuli).

- Only Elementary Tools needed for solution of problems.

Objectives

- To provide opportunities for meetings and contacts among present and future mathematicians and scientists of different countries.

- To stimulate and encourage mathematical excellence among the students and teachers

- To enrich the education and training for research in the mathematical sciences

- To foster unity of interest among all nations. Mathematics, because of its universal nature, is ideally suited for this role.

Test

- Test separated in two days.

- Each day has 3 problems. Students have 4 $\frac{1}{2}$ hours to solve the test.

Topics covered

- Number Theory
- Algebra
- Combinatorics
- Geometry

Topics Not Covered

- Calculus
- Complex Numbers*
- Inversion in geometry

Difficult Problems

The IMO has 6 very difficult questions. Usually the order of dificulty is 1, 4, 2, 5, 3, 6. Where 1,2,3 are problems on the first day and 4,5,6 on the second.

Who Competes?

- 5 Countries started.
- In 1981 it reached the 5 continents.
- IMO Mexico 2005 had 91 countries.
- Since 1983 each country is represented by 6 students under 20 years of age and that haven't started university studies.

Prizes

- Half the contestants get a medal.

- Gold Medal for the best $\frac{1}{12}$

- Silver Medal for the next $\frac{1}{6}$

- Bronze Medal for the next $\frac{1}{4}$

More Prizes

- There are also awarded honorary mentions for contestants that solve one problem, yet don't get a medal.

- On some occasions a solution is worthy of being awarded 'Creative Solution'

Example of Creative Solution

Problem 3 of IMO Mexico 2005 (July 13, 2005):

Let $x, y, z \in \mathbf{R}_0$ and $xyz \geq 1$. Prove that

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{x^2 + y^5 + z^2} + \frac{z^5 - z^2}{x^2 + y^2 + z^5} \geq 0$$

Solution

Proof.

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} - \frac{x^5 - x^2}{x^3(x^2 + y^2 + z^2)} = \frac{(x^3 - 1)^2 x^2 (y^2 + z^2)}{x^3(x^2 + y^2 + z^2)(x^5 + y^2 + z^2)} \geq 0$$

Therefore

$$\sum \frac{x^5 - x^2}{x^5 + y^2 + z^2} \geq \sum \frac{x^5 - x^2}{x^3(x^2 + y^2 + z^2)} = \frac{1}{x^2 + y^2 + z^2} \sum (x^2 - \frac{1}{x}) \geq$$

$$\geq \frac{1}{x^2 + y^2 + z^2} \sum (x^2 - yz) \geq 0$$

□

IMO Impact on Mathematics

- Timothy Gowers, Gold Medal, UK; IMO USA 1981. Fields Medal in 1998.

- Alexander Razborov, Gold Medal, USSR; IMO UK 1979. Nevanlinna Prize in 1990.

- Richard Borcherds, Silver Medal, UK; IMO Yugoslavia 1977. Fields Medal 1998.

- Peter Shor, Silver Medal, USA; IMO Yugoslavia 1977. Nevanlinna Prize 1998.

More

- Jean-Christof Yoccoz, Gold Medal, France; IMO E.Germany 1974. Fields Medal 1994.

- Vladimir Drinfel'd, Gold Medal, USSR; IMO Romania, 1969. Fields Medal 1990.

- Grigorig Margulis, Silver Medal USSR; IMO Czechoslovakia 1962. Fields Medal 1983.

Procedures

- Short-List of Problems
- Selection of Test
- Test
- Grading
- Coordinating
- Medal Cutoffs

Mexican Mathematical Olympiad (OMM)

- Started in 1987.

- Selects representatives for Mexico in the IMO.

Sample Problem from an OMM

Problem 6 of XV Mexican Mathematical Olympiad, November 2001.

A collector of rare coins has coins of denominations $1, 2, \ldots, n$ (several coins for each denomination). He wishes to put the coins into 5 boxes so that:

(1) in each box there is at most one coin of each denomination;

(2) each box has the same number of coins and the same denomination total;

(3) any two boxes contain all the denominations;

(4) no denomination is in all 5 boxes.

For which n is this possible?

Road to the IMO in Mexico

- Be top 16 in the Mexican Mathematical Olympiad (OMM) held in November.

- Every month from December to May there is a week of lectures with selection exams.

- 5 selection tests in May

- Make the team and 2 more weeks of training.

Chihuahua Mathematical Olympiad

- Started in 1989.
- Top 3 State in the nation in the history of the olympiad.
- Selects 6 students for the Mexican Mathematical Olympiad.

Road to the OMM from Chihuahua

- Be top 20 in the State.
- Be top 6 in 5 selection tests
- 1 week intensive training prior to the OMM

Evolution

The organization has evolved with time having much better prepared lecturers and different ways of selecting the representatives of the state.

Why do people care about this?

The quest for a solution is always fun, and there are few things more rewarding than a solution of a nice problem. The Olympiad has a very noble cause and it attracts many of us how want to learn.

I train young students now, because I want them to experience the joy of solving a problem and because I know that my lectures and the event can help them have new perspectives on mathematics and on life.

My Problem

I invented this problem for the Chihuahua Mathematical Olympiad in 2003:

Let A be a 21-sided regular polygon. How many isosceles triangles are formed by taking 3 vertices from A.

Experience on IMO Mexico 2005

Hear me talk.

Impact on my Life

- Discovery of the magic of mathematics.
- Getting to know interesting people around the nation.
- Opened some doors to enter selective programs (like this REU).
- Got me motivated into learning more and to try to pass this feeling towards more youngsters.

Favorite Problem

Problem 6 from an argentinian selection test to pick members for the ‘ XI Olimpiada Matematica del Cono Sur 2000’

Let P: $\mathbf{N} \Rightarrow \mathbf{N}$ be a function such that $P(1) = 1$, $P(2) = 2$ and $P(n) = P(n-1) + P(\lfloor \frac{n}{2} \rfloor)$.

Prove there exists $N > 2000$ such that $7|P(N)$

Solution

Let m be such that $P(m) \equiv 0 \mod (7)$. m exists because $P(5) = 7$. Let $P(2m-1) \equiv x \mod (7)$.

$$P(2m) = P(2m-1) + P(m) \equiv x + 0 \equiv x \mod (7)$$

$$P(2m+1) = P(2m) + P(m) \equiv x + 0 \equiv x \mod (7)$$

Let $P(4m-3) \equiv y \mod (7)$

$$P(4m-2) = P(4m-3) + P(2m-1) \equiv y + x \mod (7)$$

$$P(4m-1) = P(4m-2) + P(2m-1) \equiv y + 2x \mod (7)$$

$$P(4m) = P(4m-1) + P(2m) \equiv y + 3x \mod (7)$$

$$\vdots$$

$$P(4m+3) = P(4m+2) + P(2m+1) \equiv y + 6x \mod (7)$$