Name: ____________________

Math 25 Final Exam

May 16, 2013

Instructions: This is a closed book, closed notes exam (except for one sheet of paper). You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	40	
3	40	
4	10	
5	20	
6	20	
7	20	
8	20	
9	10	
Total:	200	

1. Answer True or False to the following statements. Please answer on the left.

   (a) [2 points] $\displaystyle\int \sin(f(x))\,dx = -\cos(f(x)) + C.$

   (b) [2 points] If f is an increasing continuous function on $[a, b]$ then $\text{LEFT}(n) \leq \displaystyle\int_a^b f(x)\,dx.$

   (c) [2 points] Integration by parts comes from "reverse-engineering" the "chain-rule" from differentiation.

   (d) [2 points] If $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converge then $\displaystyle\sum_{n=1}^{\infty}(a_n b_n)$ converges.

   (e) [2 points] If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges then $\displaystyle\sum_{n=1}^{\infty} |a_n|$ converges.

   (f) [2 points] If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

   (g) [2 points] If $0 \leq a_n \leq b_n$ and $\sum a_n$ diverges, then $\sum b_n$ diverges.

   (h) [2 points] The Taylor series for $\ln(1 - x^3)$ about $x = 0$ has only odd powers.

   (i) [2 points] If a sequence is convergent and bounded, then it must be monotone.

   (j) [2 points] $1 + x + x^2$ is a power series.

2. Integrals

   (a) [5 points] $\int (1 + x + x^2)\, dx.$

   (b) [5 points] $\int \frac{x}{x^2 + 1}\, dx.$

(c) [5 points] $\displaystyle\int e^{2x} \sin x \, dx.$

(d) [5 points] $\displaystyle\int \frac{dx}{4x^2 - 16}.$

(e) [5 points] $\displaystyle\int \sin^4(\theta)\cos^3(\theta)\,d\theta.$

(f) [5 points] $\displaystyle\int \frac{(\ln(x))^{1.3}}{x}\,dx.$

(g) [5 points] Evaluate $\displaystyle\int_1^{e^2} \frac{1}{x}\,dx$.

(h) [5 points] Calculate $\displaystyle\int_0^{3\pi} \sin(7x)\,dx$.

3. Determine whether the following series converge (and show why or why not):

   (a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.3}+12}$.

   (b) [5 points] $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$.

(c) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 - n}{(n+2)^5 - 12(n+2)^4 + 7}$.

(d) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^4}$.

(e) [5 points] $\displaystyle\sum_{n=1}^{\infty}\left(.001+\frac{1}{n^2}\right)$.

(f) [5 points] $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}+3}{n^2-7}$.

(g) [5 points] $\displaystyle\sum_{n=1}^{\infty} e^{-n^2}$.

(h) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$.

4. Find the second degree Taylor polynomial for the following functions about the given point:

   (a) [5 points] $1 + 7x + 12x^2$ about $x = -1$.

   (b) [5 points] $\dfrac{1}{x}$ about $x = 3$.

5. Find the first four nonzero terms of the Taylor series about $x = 0$ of the following functions:

   (a) [5 points] $(x^2 + 3)e^{-2x}$.

   (b) [5 points] $\dfrac{\ln(1 - x)}{1 - x}$.

(c) [5 points] $\ln(1 - x^3)$.

(d) [5 points] $\cos(2x) - \sin(2x)$.

6. Determine the interval of convergence of the following power series:

   (a) [10 points] $7 + 2x - 3x^2 + 5x^4 + x^3$.

(b) [10 points] $\sum_{n=1}^{\infty} n^2 x^n$.

7. Approximating the square root:

   (a) [5 points] Find the second degree Taylor polynomial of $\sqrt{x}$ about $x = 100$.

   (b) [5 points] Approximate $\sqrt{102}$ using (a).

(c) [10 points] Show that the approximation in (b) has an error of at most $\dfrac{1}{200000}$.

8. Using Taylor series, figure out what number the following sums converge to:

   (a) [5 points] $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$

   (b) [5 points] $1 + 2(0.5) + 3(0.5)^2 + 4(0.5)^3 + 5(0.5)^4 + \dots.$

(c) [5 points] $\frac{2}{3} + \frac{2^2}{3^2(2)} + \frac{2^3}{3^3(3)} + \frac{2^4}{3^4(4)} + \ldots$

(d) [5 points] $\frac{\pi^3}{3!} - \frac{\pi^5}{5!} + \frac{\pi^7}{7!} - \frac{\pi^9}{9!} + \ldots$

9. [10 points] Two trains 100 miles apart are moving toward each other; each one is going at a speed of 10 miles per hour. A fly starting on the front of one of the trains flies back and forth between them at a rate of 20 miles per hour. It does this until the trains collide and crush the fly to death. What is the total distance the fly has flown?