Name: ______________________________

Math 25 Final Practice Exam

May 3, 2013

Instructions: This is a closed book, closed notes exam (except for one sheet of paper). You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	40	
3	10	
4	40	
5	10	
6	20	
7	20	
8	20	
9	20	
Total:	200	

1. Answer True or False to the following statements. Please answer on the left.

   (a) [2 points] $\int f'(x)\cos(f(x))\,dx = \sin(f(x)) + C$.

   (b) [2 points] The midpoint rule is never exact.

   (c) [2 points] If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge then $\sum_{n=1}^{\infty}(a_n + b_n)$ converges.

   (d) [2 points] If a region in the xy-plane lies below the x-axis, then revolving the region around the x-axis gives a solid of negative volume.

   (e) [2 points] If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

   (f) [2 points] If $0 \le a_n \le b_n$ and $\sum a_n$ converges, then $\sum b_n$ converges.

   (g) [2 points] The Taylor series for $x^3 \cos x$ about $x = 0$ has only odd powers.

   (h) [2 points] If f has the following Taylor series about $x = 0$, then $f^{(7)}(0) = -8$:

   $$f(x) = 1 - 2x + \frac{3}{2!}x^2 - \frac{4}{3!}x^3 + \dots.$$

   (Assume the pattern of the coefficients continues.)

   (i) [2 points] The Taylor series is named after Taylor Swift.

   (j) [2 points] A Taylor series for a function $f(x)$ is a power series expansion of $f(x)$.

2. Integrals

(a) [5 points] $\int (x^{3/2} + x^{2/3})\, dx.$

(b) [5 points] $\int \frac{x^3 + x + 1}{x^2}\, dx.$

(c) [5 points] $\int \ln(x^2)\, dx$.

(d) [5 points] $\int \frac{dx}{x^2 - 9}$.

(e) [5 points] $\int \sin(5\theta) \cos^3(5\theta) \, d\theta$.

(f) [5 points] $\int \frac{(\ln(x))^2}{x} \, dx$.

(g) [5 points] $\displaystyle\int_0^2 (x^2 + x + 1)\,dx.$

(h) [5 points] $\displaystyle\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx.$

3. Volume

   (a) [5 points] What is the volume of a cone that has a circular base of radius 1 m and has height 10 m?

   (b) [5 points] The tip of the cone in part (a) is cut off 6 m from the tip. What is the volume of the truncated cone?

4. Determine whether the following series converge (and show why or why not):

   (a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+1}$.

   (b) [5 points] $\displaystyle\sum_{n=12}^{\infty} \frac{1}{(\ln n)^3(\ln(\ln(n)))}$.

(c) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(n-2)^5 + 12}$.

(d) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$.

(e) [5 points] $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n^3}\right)$.

(f) [5 points] $\displaystyle\sum_{n=1}^{\infty} n$.

(g) [5 points] $\sum_{n=1}^{\infty} e^{-n}$.

(h) [5 points] $\sum_{n=1}^{\infty} \frac{1}{\pi^2}$.

5. Find the second degree Taylor polynomial for the following functions about the given point:

   (a) [5 points] e^x about $x = -1$.

   (b) [5 points] $\ln x$ about $x = 3$.

6. Find the first four nonzero terms of the Taylor series about $x = 0$ of the following functions:

   (a) [5 points] xe^{3x}.

   (b) [5 points] $x^2 \cos(x^3)$.

(c) [5 points] $\ln(1-3x)$.

(d) [5 points] $\cos(3x)+\sin(2x)$.

7. Find the first four nonzero terms of the Taylor series about $x = 0$ and determine the interval of convergence of the following functions:

   (a) [10 points] $\dfrac{1}{1-3x}$.

(b) [10 points] $x^2 \sin(x^3)$.

8. Approximating a function and how big the error is:

   (a) [10 points] A function f has $f(3) = 1$, $f'(3) = 5$ and $f''(3) = -10$. What is the best estimate for $f(3.1)$ that can be estimated using only this information?

   (b) [10 points] Assume that $|f'''(x)| \leq 6$ for $3 \leq x \leq 3.1$. How big can the error be in the estimate of $f(3.1)$?

9. Using Taylor series, figure out what the following sums converge to:

   (a) [5 points] $1 - 2 + \frac{4}{2!} - \frac{8}{3!} + \frac{16}{4!} - \dots.$

   (b) [5 points] $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

(c) [5 points] $7 + \frac{7}{\pi} + \frac{7}{\pi^2} + \frac{7}{\pi^3} + \dots$

(d) [5 points] $\frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \dots$