2. Since $6x + x^2 = x(6 + x)$, we take

$$\frac{x+1}{6x+x^2} = \frac{A}{x} + \frac{B}{6+x}.$$

So,

$$x + 1 = A(6 + x) + Bx$$
$$x + 1 = (A + B)x + 6A,$$

giving

$$A + B = 1$$
$$6A = 1.$$

Thus $A = 1/6$, and $B = 5/6$ so

$$\frac{x+1}{6x+x^2} = \frac{1/6}{x} + \frac{5/6}{6+x}.$$

12. Using the result of Exercise 5, we have

$$\int \frac{2}{s^4-1}\,ds = \int\left(\frac{1}{2(s-1)} - \frac{1}{2(s+1)} - \frac{1}{s^2+1}\right) ds = \frac{1}{2}\ln|s-1| - \frac{1}{2}\ln|s+1| - \arctan s + C.$$

13. Using the result of Problem 6, we have

$$\int \frac{2y}{y^3-y^2+y-1}\,dy = \int \frac{1}{y-1}\,dy + \int \frac{1-y}{y^2+1}\,dy = \ln|y-1| + \arctan y - \frac{1}{2}\ln\left|y^2+1\right| + C.$$

14. Using the result of Exercise 7, we have

$$\int \frac{1}{w^4-w^3}\,dw = \int\left(\frac{1}{w-1} - \frac{1}{w} - \frac{1}{w^2} - \frac{1}{w^3}\right) dw = \ln|w-1| - \ln|w| + \frac{1}{w} + \frac{1}{2w^2} + C.$$

15. We let

$$\frac{3x^2-8x+1}{x^3-4x^2+x+6} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

giving

$$3x^2-8x+1 = A(x+1)(x-3) + B(x-2)(x-3) + C(x-2)(x+1)$$
$$3x^2-8x+1 = (A+B+C)x^2 - (2A+5B+C)x - 3A + 6B - 2C$$

so

$$A+B+C = 3$$
$$-2A-5B-C = -8$$
$$-3A+6B-2C = 1.$$

Thus, $A = B = C = 1$, so

$$\int \frac{3x^2-8x+1}{x^3-4x^2+x+6}\,dx = \int \frac{dx}{x-2} + \int \frac{dx}{x+1} + \int \frac{dx}{x-3} = \ln|x-2| + \ln|x+1| + \ln|x-3| + K.$$

We use K as the constant of integration, since we already used C in the problem.

16. We let

$$\frac{1}{x^3-x^2} = \frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

giving

$$1 = Ax(x-1) + B(x-1) + Cx^2$$
$$1 = (A+C)x^2 + (B-A)x - B$$

so

$$A+C = 0$$
$$B-A = 0$$
$$-B = 1.$$

Thus, $A = B = -1$, $C = 1$, so

$$\int \frac{dx}{x^3-x^2} = -\int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{dx}{x-1} = -\ln|x| + x^{-1} + \ln|x-1| + K.$$

We use K as the constant of integration, since we already used C in the problem.

18. Division gives

$$\frac{x^4+12x^3+15x^2+25x+11}{x^3+12x^2+11x} = x + \frac{4x^2+25x+11}{x^3+12x^2+11x}.$$

Since $x^3+12x^2+11x = x(x+1)(x+11)$, we write

$$\frac{4x^2+25x+11}{x^3+12x^2+11x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+11}$$

giving

$$4x^2+25x+11 = A(x+1)(x+11) + Bx(x+11) + Cx(x+1)$$
$$4x^2+25x+11 = (A+B+C)x^2 + (12A+11B+C)x + 11A$$

so

$$A+B+C = 4$$
$$12A+11B+C = 25$$
$$11A = 11.$$

Thus, $A = B = 1$, $C = 2$ so

$$\int \frac{x^4+12x^3+15x^2+25x+11}{x^3+12x^2+11x}\,dx = \int x\,dx + \int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{2dx}{x+11}$$
$$= \frac{x^2}{2} + \ln|x| + \ln|x+1| + 2\ln|x+11| + K.$$

We use K as the constant of integration, since we already used C in the problem.

19. Division gives

22. Since $x = \sin t + 2$, we have

$$4x - 3 - x^2 = 4(\sin t + 2) - 3 - (\sin t + 2)^2 = 1 - \sin^2 t = \cos^2 t$$

and $dx = \cos t\, dt$, so substitution gives

$$\int \frac{1}{\sqrt{4x - 3 - x^2}} = \int \frac{1}{\sqrt{\cos^2 t}} \cos t\, dt = \int dt = t + C = \arcsin(x - 2) + C.$$

32. Since $2z - z^2 = 1 - (z-1)^2$, we have

$$\int \frac{z-1}{\sqrt{2z-z^2}}\,dz = \int \frac{z-1}{\sqrt{1-(z-1)^2}}\,dz.$$

Substitute $w = 1-(z-1)^2$, so $dw = -2(z-1)\,dz$.

38. The denominator $x^2 - 3x + 2$ can be factored as $(x-1)(x-2)$. Splitting the integrand into partial fractions with denominators $(x-1)$ and $(x-2)$, we have

$$\frac{x}{x^2-3x+2} = \frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}.$$

Multiplying by $(x-1)(x-2)$ gives the identity

$$x = A(x-2) + B(x-1)$$

so

$$x = (A+B)x - 2A - B.$$

50. We have

$$\frac{10}{(s+2)(s^2+1)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+1}.$$

Thus,

$$10 = A(s^2+1) + (Bs+C)(s+2)$$
$$10 = (A+B)s^2 + (2B+C)s + (A+2C),$$

giving

$$A+B = 0$$
$$2B+C = 0$$
$$A+2C = 10.$$

Thus, from the first two equations we have $C = -2B = 2A$, which, when used in the third, gives $5A = 10$, so that $A = 2$, $B = -2$, and $C = 4$. We now have

$$\frac{10}{(s+2)(s^2+1)} = \frac{2}{s+2} + \frac{-2s+4}{s^2+1} = \frac{2}{s+2} - \frac{2s}{s^2+1} + \frac{4}{s^2+1},$$

so

$$\int \frac{10}{(s+2)(s^2+1)}\,ds = \int \left(\frac{2}{s+2} - \frac{2s}{s^2+1} + \frac{4}{s^2+1}\right) ds = 2\ln|s+2| - \ln\left|s^2+1\right| + 4\arctan s + K.$$

We use K as the constant of integration, since we already used C in the problem.

60. Using partial fractions, we write

$$\frac{2x}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$2x = A(x-1) + B(x+1) = (A+B)x - A + B.$$

So, $A + B = 2$ and $-A + B = 0$, giving $A = B = 1$. Thus

$$\int \frac{2x}{x^2-1}\,dx = \int \left(\frac{1}{x+1} + \frac{1}{x-1}\right) dx = \ln|x+1| + \ln|x-1| + C.$$

Using the substitution $w = x^2 - 1$, we get $dw = 2x\,dx$, so we have

$$\int \frac{2x}{x^2-1}\,dx = \int \frac{dw}{w} = \ln|w| + C = \ln\left|x^2-1\right| + C.$$

The properties of logarithms show that the two results are the same:

$$\ln|x+1| + \ln|x-1| = \ln|(x+1)(x-1)| = \ln\left|x^2-1\right|.$$

62. (a) We differentiate:

$$\frac{d}{d\theta}\left(-\frac{1}{\tan\theta}\right) = \frac{1}{\tan^2\theta}\cdot\frac{1}{\cos^2\theta} = \frac{1}{\frac{\sin^2\theta}{\cos^2\theta}}\cdot\frac{1}{\cos^2\theta} = \frac{1}{\sin^2\theta}.$$

Thus,

$$\int \frac{1}{\sin^2\theta}\,d\theta = -\frac{1}{\tan\theta} + C.$$

(b) Let $y = \sqrt{5}\sin\theta$ so $dy = \sqrt{5}\cos\theta\,d\theta$ giving

$$\int \frac{dy}{y^2\sqrt{5-y^2}} = \int \frac{\sqrt{5}\cos\theta}{5\sin^2\theta\sqrt{5-5\sin^2\theta}}\,d\theta = \frac{1}{5}\int \frac{\sqrt{5}\cos\theta}{\sin^2\theta\sqrt{5}\cos\theta}\,d\theta$$

$$= \frac{1}{5}\int \frac{1}{\sin^2\theta}\,d\theta = -\frac{1}{5\tan\theta} + C.$$

Since $\sin\theta = y/\sqrt{5}$, we have $\cos\theta = \sqrt{1-(y/\sqrt{5})^2} = \sqrt{5-y^2}/\sqrt{5}$. Thus,

$$\int \frac{dy}{y^2\sqrt{5-y^2}} = -\frac{1}{5\tan\theta} + C = -\frac{\sqrt{5-y^2}/\sqrt{5}}{5(y/\sqrt{5})} + C = -\frac{\sqrt{5-y^2}}{5y} + C.$$

66. (a) We integrate to find

$$\int \frac{b}{x(1-x)}\,dx = b\int\left(\frac{1}{x}+\frac{1}{1-x}\right)dx = b(\ln|x|-\ln|1-x|)+C = b\ln\left|\frac{x}{1-x}\right|+C,$$

so

$$t(p) = \int_a^p \frac{b}{x(1-x)}\,dx = b\ln\left(\frac{p}{1-p}\right) - b\ln\left(\frac{a}{1-a}\right) = b\ln\left(\frac{p(1-a)}{a(1-p)}\right).$$

(b) We know that $t(0.01)=0$ so

$$0 = b\ln\left(\frac{0.01(1-a)}{0.99a}\right).$$

But $b>0$ and $\ln x = 0$ means $x=1$, so

$$\begin{aligned}\frac{0.01(1-a)}{0.99a} &= 1\\ 0.01(1-a) &= 0.99a\\ 0.01-0.01a &= 0.99a\\ a &= 0.01.\end{aligned}$$

(c) We know that $t(0.5)=1$ so

$$1 = b\ln\left(\frac{0.5\cdot 0.99}{0.5\cdot 0.01}\right) = b\ln 99,\, b = \frac{1}{\ln 99} = 0.218.$$

(d) We have

$$t(0.9) = \int_{0.01}^{0.9} \frac{0.218}{x(1-x)}\,dx = \frac{1}{\ln 99}\ln\left(\frac{0.9(1-0.01)}{0.01(1-0.9)}\right) = 1.478.$$

68. We complete the square in the exponent so that we can make a substitution:

$$
\begin{aligned}
m(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} e^{-x^2/2}\, dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x^2-2tx)/2}\, dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-((x^2-2tx+t^2)-t^2)/2}\, dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-t)^2/2} \cdot e^{t^2/2}\, dx \\
&= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-t)^2/2}\, dx.
\end{aligned}
$$

Substitute $w = x - t$, then $dw = dx$ and $w = \infty$ when $x = \infty$, and $w = -\infty$ when $x = -\infty$. Thus

$$
\begin{aligned}
m(t) &= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-w^2/2}\, dw = \frac{e^{t^2/2}}{\sqrt{2\pi}} \cdot \sqrt{2\pi} \\
m(t) &= e^{t^2/2}.
\end{aligned}
$$