2. (a) The approximation LEFT(2) uses two rectangles, with the height of each rectangle determined by the left-hand endpoint. See Figure 7.8. We see that this approximation is an overestimate.

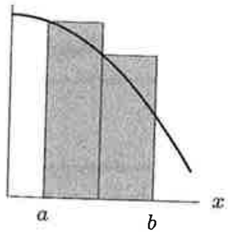


Figure 7.8

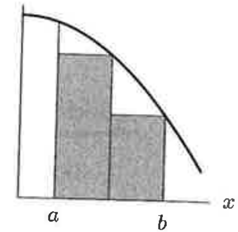


Figure 7.9

(b) The approximation RIGHT(2) uses two rectangles, with the height of each rectangle determined by the right-hand endpoint. See Figure 7.9. We see that this approximation is an underestimate.

(c) The approximation TRAP(2) uses two trapezoids, with the height of each trapezoid given by the secant line connecting the two endpoints. See Figure 7.10. We see that this approximation is an underestimate.

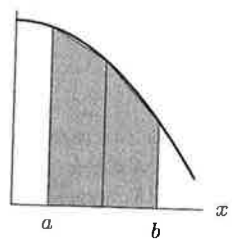


Figure 7.10

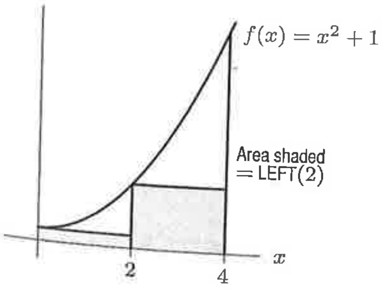
(d) The approximation MID(2) uses two rectangles, with the height of each rectangle determined by the height at the midpoint. Alternately, we can view MID(2) as a trapezoid rule where the height is given by the tangent line at the midpoint. Both interpretations are shown in Figure 7.11. We see from the tangent line interpretation that this approximation is an overestimate.

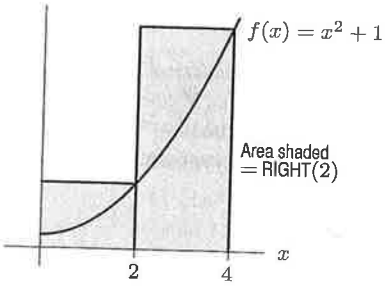
Figure 7.11

8. (a)

$$\begin{aligned} \text{LEFT}(2) &= 2 \cdot f(0) + 2 \cdot f(2) \\ &= 2 \cdot 1 + 2 \cdot 5 \\ &= 12 \\ \text{RIGHT}(2) &= 2 \cdot f(2) + 2 \cdot f(4) \\ &= 2 \cdot 5 + 2 \cdot 17 \\ &= 44 \end{aligned}$$

(b)





9. (a) LEFT(2) is an underestimate, while RIGHT(2) is an overestimate.

...creasing, so RIGHT gives an overestimate and LEFT gives an underestimate.

16. $f(x)$ is concave down, so MID gives an overestimate and TRAP gives an underestimate.

17. $f(x)$ is decreasing and concave up, so LEFT and TRAP ...

18. $f(x)$ is ...

22. (a) The graph of $y = \sqrt{2 - x^2}$ is the upper half of a circle of radius $\sqrt{2}$ centered at the origin. The integral repres the area under this curve between the lines $x = 0$ and $x = 1$. From Figure 7.29, we see that this area can be into 2 parts, A_1 and A_2. Notice since $OQ = QP = 1$, ΔOQP is isosceles. Thus $\angle POQ = \angle ROP = \frac{\pi}{4}$, and is exactly $\frac{1}{8}$ of the entire circle. Thus the total area is

$$\text{Area} = A_1 + A_2 = \frac{1}{8}\pi(\sqrt{2})^2 + \frac{1 \cdot 1}{2} = \frac{\pi}{4} + \frac{1}{2}.$$

(b) LEFT(5) ≈ 1.32350, RIGHT(5) ≈ 1.24066, T
TRAP(5) ≈ 1.28208, MID(5) ≈ 1.28705

$$\text{Exact value} \approx 1.285398163$$

Left-hand error ≈ -0.03810, Right-hand error ≈ 0.04474,
Trapezoidal error ≈ 0.00332, Midpoint error ≈ -0.001656

Thus right-hand error $>$ trapezoidal error $> 0 >$ midpoint error $>$ left-hand error, and $|$midpt error$| < |$trap erro $|$left-error$| < |$right-error$|$.

23. We approximate the area of the playing field by using Riemann sum...

2. (a) From Problem 9 on page 540, for $\int_0^4 (x^2+1)\,dx$, we have MID(2)= 24 and TRAP(2)= 28. Thus,

$$\begin{aligned}\text{SIMP}(2) &= \frac{2\text{MID}(2)+\text{TRAP}(2)}{3}\\ &= \frac{2(24)+28}{3}\\ &= \frac{76}{3}.\end{aligned}$$

(b)

$$\int_0^4 (x^2+1)\,dx = \left(\frac{x^3}{3}+x\right)\Bigg|_0^4 = \left(\frac{64}{3}+4\right)-(0+0) = \frac{76}{3}$$

(c) Error= 0. Simpson's Rule gives the exact answer.

Problems

to get our desired accuracy.

8. (a) Since LEFT is too small and RIGHT is too big, the function appears to be increasing. Since TRAP is too small and MID is too big, the function appears to be concave down.

(b) Error = Exact − Approximation = 7.621372 − Approximation. See column (b) below.

(c) Since n goes from 3 to 30, it is multiplied by 10. Thus, the errors for the LEFT and RIGHT go down by a factor of $1/10$, the errors for TRAP and MID go down by a factor of $1/10^2 = 1/100$, and the error for SIMP goes down by a factor of $1/10^4 = 1/10000$. See column (c) below.

	approximation for $n = 3$	(b) error for $n = 3$	(c) error for $n = 30$
LEFT	5.416101	2.205271	0.2205721
RIGHT	9.307921	−1.686549	−0.1686549
TRAP	7.362011	0.259361	0.0025936
MID	7.742402	−0.12103	−0.0012103
SIMP	7.615605	0.005767	0.0000006

computed using Length $\times$ Width $= 5 \cdot 3 = 15$.

2. Using similar triangles, the height, y, of the strip is given by

$$\frac{y}{3} = \frac{x}{6} \quad \text{so} \quad y = \frac{x}{2}.$$

Thus,

$$\text{Area of strip } \approx y\Delta x = \frac{x}{2}\Delta x,$$

so

$$\text{Area of region } = \int_0^6 \frac{x}{2}\,dx = \left.\frac{x^2}{4}\right|_0^6 = 9.$$

Check: This area can also be computed using the formula $\frac{1}{2}$ Base $\cdot$ Height $= \frac{1}{2} \cdot 6 \cdot 3 = 9$.

3. By similar triangles, if w is the length of the strip at height ...

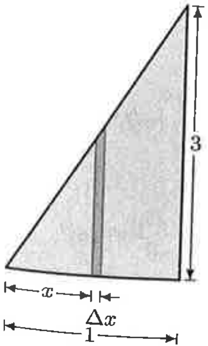


Figure 8.6

16. Semicircle of radius $r = 9$. See Figure 8.7.

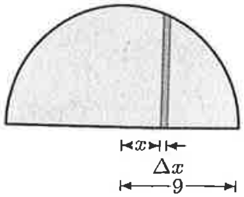


Figure 8.7