12. Each slice is a rectangular slab of length 10 m and width that decreases with height. See Figure 8.3. At height y, the l x is given by the Pythagorean Theorem

$$y^2 + x^2 = 7^2.$$

Solving gives $x = \sqrt{7^2 - y^2}$ m. Thus the width of the slab is $2x = 2\sqrt{7^2 - y^2}$ and

$$\text{Volume of slab} = \text{Length} \cdot \text{Width} \cdot \text{Height} = 10 \cdot 2\sqrt{7^2 - y^2} \cdot \Delta y = 20\sqrt{7^2 - y^2}\Delta y \text{ m}^3.$$

Summing over all slabs, we have

$$\text{Total volume} \approx \sum 20\sqrt{7^2 - y^2}\Delta y \text{ m}^3.$$

Taking a limit as $\Delta y \to 0$, we get

$$\text{Total volume} = \lim_{\Delta y \to 0} \sum 20\sqrt{7^2 - y^2}\Delta y = \int_0^7 20\sqrt{7^2 - y^2}\,dy \text{ m}^3.$$

To evaluate, we use the table of integrals or the fact that $\int_0^7 \sqrt{7^2 - y^2}\,dy$ represents the area of a quarter circle of ra 7, so

$$\text{Total volume} = \int_0^7 20\sqrt{7^2 - y^2}\,dy = 20 \cdot \frac{1}{4}\pi 7^2 = 245\pi \text{ m}^3.$$

Check: the volume of a half cylinder can also be calculated using the formula $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi 7^2 \cdot 10 = 245\pi \text{ m}^3$

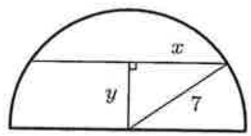


Figure 8.3

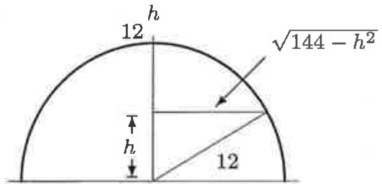


Figure 8.10

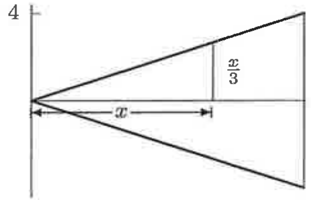


Figure 8.11

20. Cone with height 12 and radius $12/3 = 4$. See Figure 8.11.

26. (a) A vertical slice has a triangular shape and thickness Δx. See Figure 8.15.

$$\text{Volume of slice} = \text{Area of triangle} \cdot \Delta x = \frac{1}{2}\,\text{Base} \cdot \text{Height} \cdot \Delta x = \frac{1}{2} \cdot 2 \cdot 3\Delta x = 3\Delta x \text{ cm}^3.$$

Thus,

$$\text{Total volume} = \lim_{\Delta x \to 0} \sum 3\Delta x = \int_0^4 3\,dx = 3x\Big|_0^4 = 12 \text{ cm}^3.$$

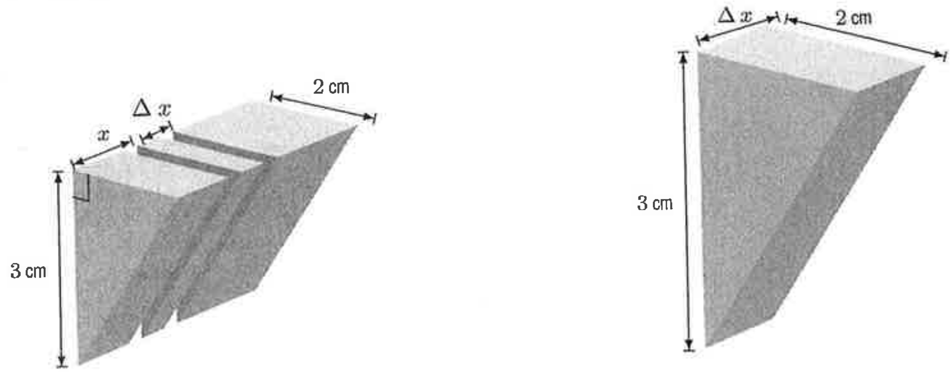


Figure 8.15

(b) A horizontal slice has a rectangular shape and thickness Δh. See Figure 8.16. Using similar triangles, we see tha

$$\frac{w}{2} = \frac{3-h}{3},$$

so

$$w = \frac{2}{3}(3-h) = 2 - \frac{2}{3}h.$$

Thus

$$\text{Volume of slice } \approx 4w\Delta h = 4\left(2 - \frac{2}{3}h\right)\Delta h = \left(8 - \frac{8}{3}h\right)\Delta h.$$

So,

$$\text{Total volume } = \lim_{\Delta h \to 0} \sum \left(8 - \frac{8}{3}h\right)\Delta h = \int_0^3 \left(8 - \frac{8}{3}h\right) dh = \left(8h - \frac{4h^2}{3}\right)\Bigg|_0^3 = 12 \text{ cm}^3.$$

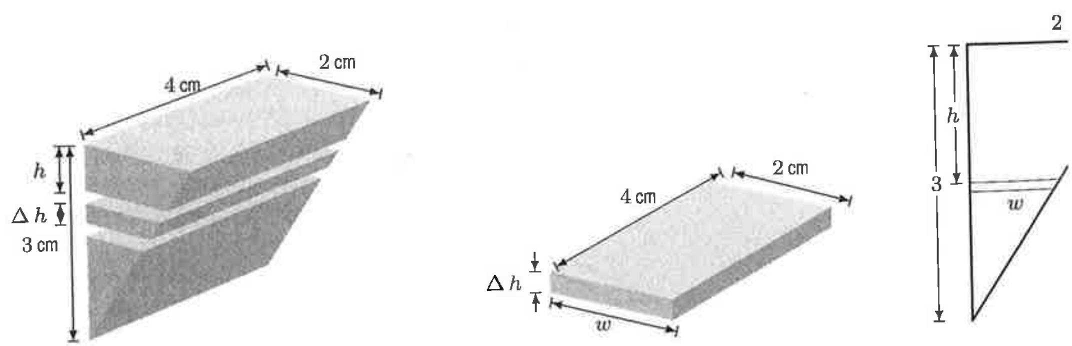


Figure 8.16

2. The volume is given by

$$V = \int_1^2 \pi y^2\,dx = \int_1^2 \pi(x+1)^4\,dx = \frac{\pi(x+1)^5}{5}\Big|_1^2 = \frac{211\pi}{5}.$$

3. The volume is given by

$$V = \int_{-2}^0 \pi(4-x^2)^2\,dx = \pi\int_{-2}^0 (16-8x^2+x^4)\,dx = \pi\left(16x - \frac{8x^3}{3} + \frac{x^5}{5}\right)\Big|_{-2}^0 = \frac{256\pi}{15}.$$

4. The volume is given by

$$V = \int_{-1}^1 \pi(\sqrt{x+1})^2\,dx = \pi\int_{-1}^1 (x+1)\,dx = \pi\left(\frac{x^2}{2}+x\right)\Big|_{-1}^1 = 2\pi.$$

5. The volume is given by

$$V = \int_{-1}^1 \pi y^2\,dx = \int_{-1}^1 \pi(e^x)^2\,dx = \int_{-1}^1 \pi e^{2x}\,dx = \frac{\pi}{2}e^{2x}\Big|_{-1}^1 = \frac{\pi}{2}(e^2 - e^{-2}).$$

6. The volume is given by

$$V = \int_0^{\pi/2} \pi y^2\,dx = \int_0^{\pi/2} \pi\cos^2 x\,dx.$$

Integration by parts gives

$$V = \frac{\pi}{2}(\cos x\sin x + x)\Big|_0^{\pi/2} = \frac{\pi^2}{4}.$$

7. The volume is given by

$$V = \int_0^1 \pi\left(\frac{1}{x+1}\right)^2 dx = \pi\int_0^1 \frac{dx}{(x+1)^2} = -\pi(x+1)^{-1}\Big|_0^1 = \pi\left(1-\frac{1}{2}\right) = \frac{\pi}{2}.$$

8. The volume is given by

$$V = \pi\int_0^1 (\sqrt{\cosh 2x})^2\,dx = \pi\int_0^1 \cosh 2x\,dx = \frac{\pi}{2}\sinh 2x\Big|_0^1 = \frac{\pi}{2}\sinh 2.$$

9. Since the graph of $y = x^2$ is below the graph of $y = x$ for $0 \le x \le 1$, the volume is given by

$$V = \int_0^1 \pi x^2\,dx - \int_0^1 \pi(x^2)^2\,dx = \pi\int_0^1 (x^2 - x^4)\,dx = \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \frac{2\pi}{15}.$$

10. Since the graph of $y = e^{3x}$ is above the graph of $y = e^x$ for $0 \le x \le 1$, the volume is given by

$$V = \int_0^1 \pi(e^{3x})^2\,dx - \int_0^1 \pi(e^x)^2\,dx = \int_0^1 \pi(e^{6x} - e^{2x})\,dx = \pi\left(\frac{e^{6x}}{6} - \frac{e^{2x}}{2}\right)\Big|_0^1 = \pi\left(\frac{e^6}{6} - \frac{e^2}{2} + \frac{1}{3}\right).$$

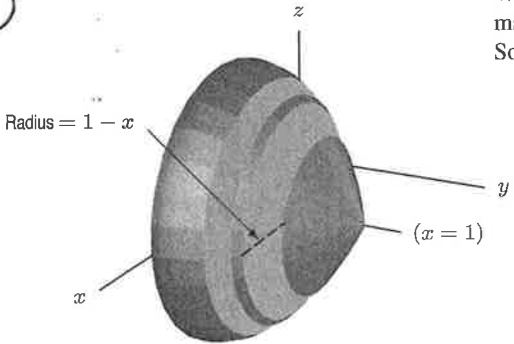
11. Since $f'(x) = x$, we evaluate the integral numerically or using the table to get

$$\text{Arc length } = \int_0^2 \sqrt{1+x^2}\,dx = \frac{\ln(\sqrt{5}+2)}{2} + \sqrt{5} = 2.958.$$

12. Since $f'(x) = -\sin x$, we evaluate the integral numerically to get

$$\text{Arc length } = \int_0^2 \sqrt{1+\sin^2 x}\,dx = 2.508.$$

30.



We slice the region perpendicular to the y–axis. The Riemann sum we get is $\sum \pi(1-x)^2 \Delta y = \sum \pi(1-y^2)^2 \Delta y$. So the volume V is the integral

$$\begin{aligned} V &= \int_0^1 \pi(1-y^2)^2\,dy \\ &= \pi \int_0^1 (1-2y^2+y^4)\,dy \\ &= \pi\left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right)\Bigg|_0^1 \\ &= (8/15)\pi \approx 1.68. \end{aligned}$$

34. The region is cylindrical with a hole around the axis of rotation, $y = -2$. Slice it into rings vertically, as in Figure 8.25. A typical ring has thickness Δx and outer radius $1 + 2 = 3$ and inner radius $y + 2 = x^2 + 2$. Thus

$$\text{Volume of slice} \approx \pi 3^2 \Delta x - \pi(x^2+2)^2 \Delta x = \pi(5 - x^4 - 4x^2)\,\Delta x.$$

$$\text{Volume of solid} = \int_0^1 \pi(5 - x^4 - 4x^2)\,\Delta x = \pi\left(5x - \frac{x^5}{5} - \frac{4}{3}x^3\right)\Bigg|_0^1 = \frac{52\pi}{15}.$$

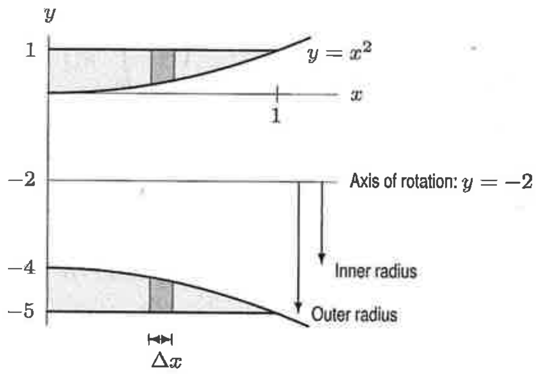


Figure 8.25: Cross-section of solid

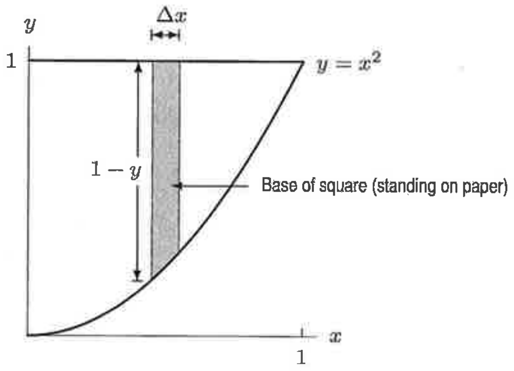


Figure 8.26: Base of solid