2. We compare with positive powers of 2, which are $2, 4, 8, 16, 32, \ldots$. Each term is one less, so we take $s_n = 2^n - 1$.

so the sequence converges to 3.

16. Since $\lim_{n\to\infty} x^n = 0$ if $|x| < 1$ and $|-0.3| < 1$, we have $\lim_{n\to\infty}(-0.3)^n = 0$, so the sequence converges to 0.

17. We have:

$$\lim_{n\to\infty}\left(\frac{n}{10}+\frac{10}{n}\right) = \lim_{n\to\infty}\frac{n}{10} + \lim_{n\to\infty} 10n.$$

Since $n/10$ gets arbitrarily large and $10/n$ approaches 0 as $n \to \infty$, the sequence diverges.

18. Since $\lim_{n\to\infty} x^n = 0$ if $|x| < 1$ and $\left|\frac{2}{3}\right| < 1$, we have $\lim_{n\to\infty}\left(\frac{2^n}{3^n}\right) = \lim_{n\to\infty}\left(\frac{2}{3}\right)^n = 0$, so the sequence conve

19. We have

$$\lim_{n\to\infty}\frac{2n+1}{n} = \lim_{n\to\infty}\left(2+\frac{1}{n}\right) = 2,$$

so the sequence converges to 2.

20. We have:

$$\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0.$$

The terms of the sequence alternate in sign, but they approach 0, so the sequence converges to 0.

21. Since the exponential function 2^n dominates the power function n^3 as $n \to \infty$, the series diverges.

22. As n increases, the term $2n$ is much larger in magnitude than $(-1)^n 5$ and the term $4n$ is much larger in magn $(-1)^n 3$. Thus dividing the numerator and denominator by n and using the fact that $\lim_{n\to\infty} 1/n = 0$, we have

$$\lim_{n\to\infty}\frac{2n+(-1)^n 5}{4n-(-1)^n 3} = \lim_{n\to\infty}\frac{2+(-1)^n 5/n}{4-(-1)^n 3/n} = \frac{1}{2}.$$

Thus, the sequence converges to $1/2$.

23. Since $\lim_{n\to\infty} 1/n = 0$ and $-1 \le \sin n \le 1$, the terms approach zero and the sequence converges to 0.

24. Since $s_n = \cos(\pi n) = 1$ if n is even and $s_n = \cos(\pi n) = -1$ if n is odd, the values of s_n alternate between 1 so the limit does not exist. Thus, the sequence diverges.

25. (a) matches (IV), since the sequence increases toward 1.
(b) matches (III), since the odd terms increase toward 1 and the even terms decrease toward 1.
(c) matches (II), since the sequence decreases toward 0.
(d) matches (I), since the sequence decreases toward 1.

26. (a) matches (II), since $\lim_{n\to\infty}(n(n+1)-1) = \infty$.
(b) matches (III), since $\lim_{n\to\infty}(1/(n+1)) = 0$ and $1/(n+1)$ is always positive.
(c) matches (I), since $\lim_{n\to\infty}(1-n^2) = -\infty$.
(d) matches (IV), since $\lim_{n\to\infty}\cos(1/n) = \cos 0 = 1$.
(e) matches (V), since $\sin n$ is bounded above and below by ± 1, so $\lim_{n\to\infty}((\sin n)/n) = 0$ and the sign of sin as $n \to \infty$.

the even positive integers and odd positive integers have the same recursive definition except for the starting term.

40. Each term is twice the previous term minus one, so a recursive definition is $s_n = 2s_{n-1} - 1$ for $n > 1$ and $s_1 = 3$. We also notice that the differences of consecutive terms are powers of 2, so $s_2 = s_1 + 2$, $s_3 = s_2 + 2^2$, and so on. Thus another recursive definition is $s_n = s_{n-1} + 2^{n-1}$ for $n > 1$ and $s_1 = 3$.

41. The differences between consecutive terms are 4, 9, 16, 25, so, for example, $s_2 = s_1 + 4$ and $s_3 = s_2 + 9$. Thus a possible

the total sales in the n months starting from January 1, 2004.

52. (a) Since you have two parents and four grandparents, $s_1 = 2$ and $s_2 = 4$. In general, $s_n = 2^n$.
(b) Solving $s_n = 6 \cdot 10^9$ gives

$$2^n = 6 \cdot 10^9$$
$$n = \frac{\ln(6 \cdot 10^9)}{\ln 2} = 32.482.$$

Thus, 33 or more generations ago, the number of ancestors is greater than the current population of the
the population of the world 33 generations ago was much smaller than it is now, there must have been ov
our ancestors.

53. In year 1, the payment is

Solutions for Section 9.2

Exercises

1. Yes, $a = 5$, ratio $= -2$.

2. No. Ratio between successive terms is not constant: $\dfrac{1/3}{1/2} = 0.66\ldots$, while $\dfrac{1/4}{1/3} = 0.75$.

3. Yes, $a = 2$, ratio $= 1/2$.

4. Yes, $a = 1$, ratio $= -1/2$.

5. No. Ratio between successive terms is not constant: $\dfrac{2x^2}{x} = 2x$, while $\dfrac{3x^3}{2x^2} = \dfrac{3}{2}x$.

6. Yes, $a = 1$, ratio $= 2z$.

7. No. Ratio between successive terms is not constant: $\dfrac{6z^2}{3z} = 2z$, while $\dfrac{9z^3}{6z^2} = \dfrac{3}{2}z$.

8. Yes, $a = 1$, ratio $= -x$.

9. Yes, $a = 1$, ratio $= -y^2$.

10. Yes, $a = y^2$, ratio $= y$.

11. The series has 26 terms. The first term is $a = 2$ and the constant ratio is $x = 0.1$, so

$$\text{Sum} = \frac{a(1-x^{26})}{(1-x)} = \frac{2(1-(0.1)^{26})}{0.9} = 2.222.$$

12. The series has 10 terms. The first term is $a = 0.2$ and the constant ratio is $x = 0.1$, so

$$\text{Sum} = \frac{0.2(1-x^{10})}{(1-x)} = \frac{0.2(1-(0.1)^{10})}{0.9} = 0.222.$$

16. Using the formula for the sum of an infinite geometric series,

$$\sum_{n=4}^{\infty}\left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5 + \cdots = \left(\frac{1}{3}\right)^4\left(1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \cdots\right) = \frac{(\frac{1}{3})^4}{1-\frac{1}{3}} = \frac{1}{54}$$

34. The amount of additional income generated directly by people spending their extra money is $\$100(0.8) =$ This additional money in turn is spent, generating another $(\$100(0.8))\,(0.8) = \$100(0.8)^2$ million. Tl indefinitely, resulting in

$$\text{Total additional income} = 100(0.8) + 100(0.8)^2 + 100(0.8)^3 + \cdots = \frac{100(0.8)}{1-0.8} = \$400 \text{ milli}$$