Name: ______________________________

Math 25 Midterm #1

February 26, 2013

Instructions: This is a closed book, closed notes exam (except for the allowed cheat sheet). You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	10	
2	40	
3	10	
4	35	
5	20	
6	20	
7	20	
8	20	
Total:	175	

1. Answer True or False to the following statements. Please answer on the left.

   (a) [2 points] The midpoint rule approximation to $\int_0^1 (y^2-1)\,dy$ is always smaller than the exact value of the integral.

   (b) [2 points] Integration by parts comes from "reverse-engineering" the "chain-rule" from differentiation.

   (c) [2 points] When choosing u in integration by parts, it is best to choose and exponential before choosing a logarithm.

   (d) [2 points] $\arctan(1) = \dfrac{\pi}{4}$.

   (e) [2 points] The trapezoid estimate $\text{TRAP}(n)$ is always closer to $\int_2^6 f(x)\,dx$ than $\text{LEFT}(n)$ or $\text{RIGHT}(n)$.

2. Solve the following indefinite integrals:

   (a) [5 points] $\int r^2 e^{r^3}\, dr.$

   (b) [5 points] $\int x^7 \ln(x)\, dx.$

(c) [5 points] $\int \sin(7x)\,dx.$

(d) [5 points] $\int x^2 e^{3x}\,dx.$

(e) [5 points] $\int y(y^2+5)^8 \, dy$.

(f) [5 points] $\int \frac{1}{\sqrt{16-9x^2}} \, dx$.

(g) [10 points] $\displaystyle\int \frac{x^3 + 12x + 7}{x^2 + 5x + 4}\, dx.$

3. Definite integrals.

   (a) [5 points] Evaluate $\displaystyle\int_2^4 \frac{x^2+1}{x-1}\,dx$.

   (b) [5 points] Show that

$$\int_1^{e^{10}} \frac{\sin t}{t^3}\,dt = \int_0^{10} \sin\left(e^u\right)e^{-2u}\,du.$$

4. Let $f(x) = |x|^{1/3}$, i.e., the absolute value of x raised to the one third.

   (a) [5 points] Set up the integral to compute the area of the region between the x-axis, $f(x)$, $x = -1$ and $x = 1$.

   (b) [5 points] Find the area of the region described in (a).

(c) [5 points] Find RIGHT(1) for the integral set up in (a).

(d) [5 points] Find MID(1) for the integral set up in (a).

(e) [5 points] Find the error for RIGHT(1).

(f) [5 points] Find the error for MID(1).

(g) [5 points] Which of RIGHT(1) and MID(1) had the smaller error?

5. Leslie wants to make the perfect Napolitan ice cream scoop for her best friend Anne. She has a spherical container of radius 5cm and the container has marks at 3cm from the bottom and at 6cm from the bottom.

   (a) [5 points] Calculate the volume of the container from the bottom to the 3cm mark.

   (b) [5 points] Calculate the volume of the container from the 3cm mark to the 6cm mark.

(c) [5 points] Calculate the volume of the container from the 6cm mark to the top.

(d) [5 points] Leslie knows Anne ranks the Napolitan flavors in the following way: 1) vanilla, 2) strawberry, 3) chocolate. How should Leslie make the scoop, i.e., what flavor goes at the bottom, what flavor in the middle and what flavor in the top so that the flavor with maximum volume is vanilla, with second highest is strawberry and with lowest is chocolate.

6. [20 points] Find the volume of the region bounded by $y = x^2$, the y-axis and $y = 16$ and rotated around the y-axis.

7. Arc Length.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1/2	$-\sqrt{2}/2$	$-\sqrt{3}/2$	-1
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\sqrt{3}/3$	0

Table 1: Values of trigonometric functions for different angles.

(a) [10 points] Set up the integral to find the arc length of the curve $y = \cos x$ where $0 \leq x \leq \pi$.

(b) [10 points] Use the trapezoidal rule with $n = 6$ to evaluate the integral you found in part (a).

8. Let $l(x) = \displaystyle\int_e^x \frac{1}{\log t}\, dt$. We want to approximate $l(10^6 + e)$ (with e being the natual number) with a left Riemann sum. Let

$$E_L(n) = \left| \int_e^{10^6+e} \frac{1}{\log t}\, dt - \text{LEFT}(n) \right|.$$

If $K_L = \max\{|f'(x)| \mid \text{ for } e \leq x \leq 10^6 + e\}$, then we know

$$E_L(n) \leq \frac{K_L(b-a)^2}{2n}.$$

(a) [5 points] Find K_L.

(b) [5 points] Give an upper bound for $E_L(n)$ (using the formula above).

(c) [5 points] Find the smallest n that guarantees that LEFT(n) is within 1000 of $l(10^6 + e)$

(d) [5 points] Find the smallest n that guarantees that LEFT(n) is within 100 of $l(10^6 + e)$.