NAME:____________________________

MATH 25 MIDTERM #2

April 2, 2013

INSTRUCTIONS: This is a closed book, closed notes exam (except for the allowed cheat sheet). You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	20	
3	20	
4	24	
5	40	
6	20	
7	30	
Total:	174	

1. Answer True or False to the following statements. Please answer on the left.

(a) [2 points] If a sequence s_n is convergent, then the terms s_n tend to zero as $n \to \infty$.

(b) [2 points] Let $f(x)$ be a positive continuous function for $x > 0$ and suppose $\int_0^\infty f(x)\,dx$ converges. Let a be a positive real number, then $\int_0^\infty f(a+x)\,dx$ converges.

(c) [2 points] A monotone sequence can not have both positive and negative terms.

(d) [2 points] If $0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges.

(e) [2 points] If a series converges, then the sequence of partial sums of the series also converges.

(f) [2 points] If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ converges.

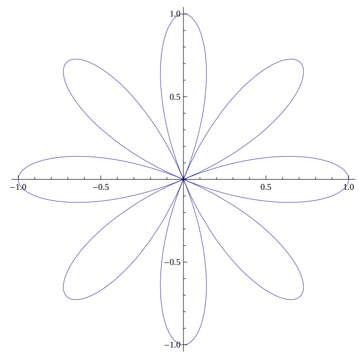
(g) [2 points] If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge then $\sum_{n=1}^{\infty} a_n b_n$ diverges.

(h) [2 points] The geometric series that has first term 1 and ratio 1.00001 diverges.

(i) [2 points] 5, 11, 17, 23 is an arithmetic progression.

(j) [2 points] 5, 11, 17, 23 is a geometric progression.

2. The figure below, which we will refer to as the "flower", is the graph of the Polar equation $r = \cos(4\theta)$ for $0 \leq \theta \leq 2\pi$.



(a) [5 points] Set up the integral that expresses the area of the "flower".

(b) [5 points] Find the area of "flower".

(c) [5 points] Set up the integral that expresses the perimeter of the "flower".

(d) [5 points] Show that the perimeter of the "flower" is greater than 16.

3. Calculate the following sums

   (a) [5 points]

$$1+2+3+5+6+7+9+10+11+13+14+15+\ldots+3997+3998+3999.$$

   (b) [5 points]

$$12 - 4 + \frac{4}{3} - \frac{4}{9} + \frac{4}{27} - \ldots.$$

(c) [5 points]

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{4^3} + \ldots.$$

(d) [5 points]

$$1 + 2 + 3 + \ldots + n.$$

4. For each of the following sequences answer whether the sequence is MONOTONE or not, whether the sequence is BOUNDED or not and whether the sequence CONVERGES or diverges..

   (a) [6 points]

   $$s_n = 1 + e^{-n}.$$

   (b) [6 points]

   $$s_n = (-1)^n.$$

   (c) [6 points]

   $$s_n = \frac{1}{n^{.00001}}.$$

   (d) [6 points]

   $$s_n = \frac{1}{7^n}$$

   .

5. Decide whether the following series converge or diverge and EXPLAIN why.

   (a) [5 points]

$$\sum_{n=1}^{\infty} 1 + e^{-n}.$$

   (b) [5 points]

$$\sum_{n=1}^{\infty} (-1)^n.$$

(c) [5 points]

$$\sum_{n=1}^{\infty} \frac{1}{n^{.00001}}.$$

(d) [5 points]

$$\sum_{n=1}^{\infty} \frac{1}{7^{n}}.$$

(e) [5 points]

$$\sum_{n=3}^{\infty} \frac{1}{n \ln(n) \ln(\ln(n))}.$$

(f) [5 points]

$$\sum_{n=1}^{\infty} \frac{n^3 - n^2 + n + 7}{n^6 + n^5 - n^4 - 32n^3 + 7}.$$

(g) [5 points]

$$\sum_{n=1}^{\infty} \frac{12}{\sqrt{7n^3 + 1984}}.$$

(h) [5 points]

$$\sum_{n=1}^{\infty} \frac{1}{n!},$$

where $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1.$

6. A ball is dropped from a height of 10 feet and bounces. Each bounce is $\frac{3}{4}$ of the height of the bounce before. Thus after the ball hits the floor for the first time, the ball rises to a height of $(10)\left(\frac{3}{4}\right)$ feet. Assume there is no air resistance.

   (a) [5 points] Find an expression for the height h_n to which the ball rises after it hits the floor for the n-th time (for example $h_1 = 7.5$ feet).

   (b) [5 points] Let d_n be the total vertical distance traveled when the ball hits the floor for the n-th time. For example $d_1 = 10$ feet, since the ball traveled 10 feet before it hit the ground. $d_2 = 25$ because the ball traveled 10 feet down, then 7.5 feet up, then 7.5 feet down, so the ball traveled $10 + 7.5 + 7.5 = 25$ feet. Find d_5.

(c) [5 points] Find a formula for d_n.

(d) [5 points] Find $\lim_{n\to\infty} d_n$.

7. Finding a formula for the sum of consecutive squares.

(a) [5 points] Show that for any positive integer n,

$$(n+1)^3 - n^3 = 3n^2 + 3n + 1.$$

(b) [5 points] Explain why the following is true:

$$\begin{array}{rcc}
(n+1)^3 - n^3 & = & 3n^2 + 3n + 1 \\
n^3 - (n-1)^3 & = & 3(n-1)^2 + 3(n-1) + 1 \\
(n-1)^3 - (n-2)^3 & = & 3(n-2)^2 + 3(n-2) + 1 \\
\vdots & \vdots & \vdots \\
3^3 - 2^3 & = & 3(2^2) + 3(2) + 1 \\
2^3 - 1^3 & = & 3(1^2) + 3(1) + 1.
\end{array}$$

(c) [5 points] Show that

$$((n+1)^3-n^3)+(n^3-(n-1)^3)+\ldots+(3^3-2^3)+(2^3-1^3) = (n+1)^3-1.$$

(d) [5 points] By summing all the rows in (b) and using (c) conclude that

$$(n+1)^3-1 = 3(1^2+2^2+\ldots+n^2)+3(1+2+\ldots+n)+n$$

.

(e) [5 points] From (d) use algebra to show:

$$1^2 + 2^2 + \ldots + n^2 = \frac{\left((n+1)^3 - 1 - 3\frac{n(n+1)}{2} - n\right)}{3}.$$

(f) [5 points] From(e) use algebra to conclude:

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

BONUS (6 points total)

1. In class we had an experiment where we threw toothpicks at a piece of paper and calculated the probability that the toothpick touches a vertical line. The problem is called __________'s Needle Problem. Fill in the blank.

2. On Women's Day we studied a curve named "The Witch of __________". Fill in the blank with the last name of an important female Italian mathematician.

3. Name 5 mathematicians (other than Swarthmore professors).