

NAME: _____

MATH 25 MIDTERM #1

February 21, 2012

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	40	
3	20	
4	20	
5	20	
6	20	
7	20	
Total:	160	

1. Answer True or False to the following statements. Please answer on the left.

(a) [2 points] $\int x \sin(5 - x^2) dx$ can be evaluated using substitution.

(b) [2 points] If f is continuous for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_a^\infty f(x) dx$ for all positive a .

(c) [2 points] If f is continuous and positive for $x > 0$ and if $\lim_{x \rightarrow \infty} f(x) = \infty$, then $\int_0^\infty \frac{dx}{f(x)}$ converges.

(d) [2 points] If $\int_0^\infty f(x) dx$ and $\int_0^\infty g(x) dx$ both diverge, then $\int_0^\infty (f(x) + g(x)) dx$ diverges.

(e) [2 points] If f is a continuous function on $[2, 6]$, then $LEFT(n) \leq \int_2^6 f(x) dx \leq RIGHT(n)$.

(f) [2 points] If $LEFT(n) = RIGHT(n)$ for all n , then f is a constant function.

(g) [2 points] The integral $\int_{-3}^3 \pi(9 - x^2) dx$ represents the volume of a sphere of radius 3.

(h) [2 points] If a region in the xy -plane lies below the x -axis, then revolving that region around the x -axis gives a solid of negative volume.

(i) [2 points] The integral $\int_0^h \pi(r - y) dy$ gives the volume of a cone of radius r and height h .

(j) [2 points] The polar equation $r = \frac{1}{\cos \theta + 3 \sin \theta}$ represents a line.

2. Solve the following indefinite integrals:

(a) [5 points] $\int e^{2r} dr.$

(b) [5 points] $\int 3^x dx.$

(c) [5 points] $\int (x^3 - 1)^4 x^2 dx.$

(d) [5 points] $\int \ln(x^2) dx.$

(e) [5 points] $\int \frac{(1 + \ln x)^2}{x} dx.$

(f) [5 points] $\int \frac{1}{\sqrt{9 - 4x^2}} dx.$

(g) [5 points] $\int \frac{12x + 7}{x^2 + 3x + 2} dx.$

(h) [5 points] $\int x(\sec^2(x)) dx.$

3. Evaluate the following definite integrals.

(a) [10 points] $\int_1^3 \frac{x^2 + 1}{x} dx.$

(b) [10 points] $\int_0^{10} xe^{-x} dx.$

4. Decide if the following integrals converge or diverge and if they converge, find its value or give a bound on its value.

(a) [10 points] $\int_{10}^{\infty} \frac{1}{x \ln x} dx.$

(b) [10 points] $\int_{-5}^{10} \frac{1}{\sqrt{x+5}} dx.$

5. Volume

(a) [10 points] What is the volume of a pyramid of height 10 m and square base 8 m by 8 m?

(b) [10 points] The pyramid in part (a) is cut off at height 6 m. What is its volume?.

6. [20 points] Find the volume of the region bounded by $y = x^2 + 1$, the x -axis, $x = 0$, $x = 4$ and rotated around the x -axis.

7. Arc Length in Polar Coordinates.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$
$\sin \theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	-1	$-\sqrt{3}/2$	$-\sqrt{2}/2$	1/2
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$
$\tan \theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞	$-\infty$	$-\sqrt{3}$	-1	$-\sqrt{3}/3$

Table 1: Values of trigonometric functions for different angles.

- (a) [10 points] Set up the integral to find the arc length of part of the cardioid $r = 1 + \cos(\theta)$ where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.
- (b) [10 points] Use the trapezoidal rule with $n = 6$ to evaluate the integral you found in part (a).